MAGNON-PHONON INTERACTION AND RENORMALIZATION OF COLLECTIVE MODES IN MAGNETIC SYSTEMS

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Sharda Prasad Saxena

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S Y N O P S I S

The physical properties of a crystal are governed by the low-lying elementary excitations in the system. In insulating crystals the most important elementary excitations are lattice vibrational modes. The acoustic modes are particularly important in low temperature regions. In the quantal description of lattice waves the crystal contains an assembly of independent harmonic oscillators which exist in certain quantum states of excitations. These excitations are likened to quasi-particles and are referred to as phonons characterised by a definite energy and a definite direction of propagation.

If the crystal is magnetically ordered also i.e. it contains an assembly of paramagnetic ions with their magnetic moments coupled in a particular fashion we have some additional collective modes. These are called magnons, the quanta associated with spin wave modes. At very low temperatures, magnons too have well-defined energy and definite direction of propagation. However, there are various interaction processes involving these elementary excitations. They can be broadly represented by phonon-phonon, magnon-magnon and magnon-phonon interaction terms in the Hamiltonian. The interaction mechanisms for these processes have application to low temperature transport properties of magnetically ordered solids. These processes involve collective modes.

In the present work, the effect of magnon-phonon interactions on the energy spectrum of the collective modes has been studied using the Green Function Technique.

In chapter 1 the fundamental concepts of quasi-particles in magnetically ordered solids such as magnons and phonons and their interactions have been discussed. Then the powerful method of Green's function is reviewed.

In chapter 2 starting with appropriate Hamiltonian involving pure phonon, pure magnon and magnon-phonon interaction terms for ferromagnet, expressions have been obtained for the renormalised phonon and magnon mode energies as poles of respective Green's functions on decoupling the Green function hierarchy at appropriate stages. Expressions have also been derived for life time of corresponding quasi-particles.

In chapter 3, expressions for renormalised mode energies and life-time of quasi-particles in the case of antiferromagnets have been derived using the same methods.

In chapter 4, the renormalised energies have been estimated for typical ferromagnetic (Ni) and antiferromagnetic (MnF_2) systems and results compared with experimental evidence. This comparison vindicates Heisenberg-Bloch model for ferromagnetism and shows that two magnon-modes are dynamically different for antiferromagnets.

Chapter 5 comprises appendices giving mathematical details of methods of integration required for solution of decoupled Green's function equations of chapters 2 and 3.

CHAPTER - 1

INTRODUCTION

1. MAGNETIC ORDER:

Ordered magnetic behaviour is now known to be a widely occurring "co-operative" phenomenon characterising an interacting system of indistinguishable particles obeying Fermi-Dirac statistics, as required by Pauli's exclusion principle. For understanding the behaviour of magnetic materials, several theoretical models have been suggested, e.g. molecular field, Ising chain, effective exchange, anisotropic exchange, etc. Attempts have been made, of late, to advance an itinerant model for the so-called band magnetisation in magnetic metals. However, the most frequently used model is based on Heisenberg Exchange Hamiltonian for localized magnetic moments (spins) in magnetic insulators and metals.

The fundamental concept of spin exchange emanates from Heisenberg's theory[1] propounded in 1928. This extension of Heitler-London's theory of chemical bond proved to be a remarkable contribution toward understanding ferromagnetism. In this model, a certain number of unpaired electrons are assumed to be present in each atom, regularly spaced in a crystal. The electronic spins are assumed to be localised at their respective sites. Heisenberg showed that an exchange effect leading to a strong-spin dependent coupling between the electrons (of spin 1/2) can be caused by ordinary Coulomb interaction, if one takes into account the exclusion principle and the quantum mechanical indistinguishability of identical particles in question. The spin-ordering is thus a dynamical consequence of this spin-correlated Coulomb interaction.

For the mathematical treatment of the above many-electron problem, two methods were suggested in 1929 - (i) the determinantal method of Slater,[2] and (ii) the spin-operator method of Dirac.[3] Dirac examined closely the concept of permutation or exchange (P_{ij}) of indistinguishable particles as a dynamical variable and the associated operator,

$$\hat{P}_{ij} = -\frac{1}{2} \left(1 + 4\hat{S}_i \cdot \hat{S}_j \right),$$

which commutes with the Hamiltonian (i.e. a q-number). His studies[3-5] led to a term in the Hamiltonian

$$H_{ex} = -\sum_{ij} J_{ij}\, \hat{\underline{S}}_i \cdot \hat{\underline{S}}_j \quad , \qquad (1.1)$$

where $\underline{S}_i$, $\underline{S}_j$ are the spin operators for the electrons in the orbitals i, j and J_{ij} is the effective exchange integral for the pair of orbitals. The above is strictly true for the spin 1/2 case only and is the most general expression for the isotropic exchange. This has, however, been extended kinematically for arbitrary spin values. A rigorous generalisation of the above exchange operator for arbitrary spin S was given by Schrodinger.

Let ψ_a and ψ_b be the localised atomic orbitals being the solutions of

$$\left(\frac{P_i^2}{2m} - \frac{Ze^2}{r_{ia}} \right) \psi_a(i) = E_a \psi_a(i) \qquad\qquad (1.2)$$
$$\left(\frac{P_j^2}{2m} - \frac{Ze^2}{r_{jb}} \right) \psi_b(j) = E_b \psi_b(j),$$

where $P_i^2/2m$, $P_j^2/2m$ are the kinetic energy operators for the electrons i and j, m is the mass of the electron, Ze is the charge of the ion core and $-Ze^2/r_{ia}$, $-Ze^2/r_{jb}$ are the potential energies; r_{ia} and r_{jb} being the distances of the electron i from the ion a and the electron j from the ion b respectively. E_a and E_b are the corresponding energies.

In terms of these wave-functions, the exchange integral J_{ij}, in the general case, is given by

$$J_{ij} = \left\langle ab \left| \frac{e^2}{r_{ij}} \right| ba \right\rangle - 2S_{ab} \langle a | V | b \rangle , \qquad (1.3)$$

where

$$V = - \frac{Ze^2}{r_{ib}} - \frac{Ze^2}{r_{ja}}$$

and

$$S_{ab} = \langle a/b \rangle = \int \psi_a^{*} \psi_b \, d\tau$$

If the orbitals are orthogonal, the overlap integral S_{ab} is zero and only the fi st term in the equation (1.3) survives. This term is always positive being the self energy of the overlap charge $e\psi^{*}(i)\,\psi b(i)$. This favours ferromagnetism. In case the orbitals are non-orthogonal, the sign of J_{ij} would depend on whether the first or the second term would dominate. When the second term dominates the antiferromagnetic coupling is favoured. In this type of crystal the total magnetic moment is zero due to anti-parallel alignment. There also exist systems in nature in which sub-lattices with antiparallel spins have a finite resultant magnetic moment due to one or more of the following factors viz. - (i) unequal spins, (ii) unequal g-factors or (iii) unequal number of sites in different sub-lattices. These magnetic systems are classified as ferrimagnets. In the present work, however, our attention is directed only towards ferromagnetic and antiferromagnetic systems.

These magnetic materials have a definite spin-ordering in the ground state which is realised only at the Absolute Zero of temperature. If the temperature is slighly increased, excitations in the spin system will be produced over the ground state. The physical properties of the system are governed by these low-lying excitations. A knowledge of the energy spectrum of the elementary excitations of a particular system can give us a clear idea of its physical behaviour. To understand this physical behaviour, one of the most powerful physical tools is the Green's function method used in conjunction with the spin-wave concept outlined below.

B. MAGNONS:

Bloch[6] conceived of a spin wave as a single spin reversal in an otherwise ordered system which, due to strong exchange interactions, does not remain localised but gets coherently distributed over the crystal lattice. Bloch showed that the low energy excited states of a ferromagnet would be of this nature. Bloch did not consider interaction between various spin waves or with lattice waves.

An altogether new technique was suggested by Holstein and Primakoff[7] (HP) to include the spin wave interactions which was followed by a rigorous and satisfying treatment by Dyson.[8] They defined a set of co-ordinates which have the appearance of spin wave amplitudes and which accurately describe the quantum state of the system.

Let us consider a body centred ferromagnet. Let S be the magnitude of the spin and $\underline{S}_1$ the spin operator for the site l. Thus it is convenient to introduce the new operators in terms of x, y and z components of the operators S_l by

$$\left.\begin{aligned} S_l^{\pm} &= S_l^{x} \pm i\, S_l^{y} \\ n_l &= S - S_l^{z} \end{aligned}\right\} \qquad (1.4)$$

The eigenstates of the operators n_l and S_l^z is written as

$$\psi_{n_1 \ldots n_l \ldots n_N} = \psi_{n_1} \qquad (1.5)$$

The corresponding eigenvalues being n_l and m_l respectively m_l takes the values S, S-1 . . . . - S. n_l takes integral values 0, 1, ... 2S, and obviously represents the difference between the z-component of the spin at the l^{th} site and its maximum value. This is known as spin deviation.

The operators defined in (1.4) have the properties:

$$\left.\begin{aligned} S_l^{+}\psi_{n_l} &= (2S)^{1/2}\left(1-\frac{n_l-1}{2S}\right)^{1/2}(n_l)^{1/2}\,\psi_{n_l-1} \\ S_l^{-}\psi_{n_l} &= (2S)^{1/2}(n_l+1)^{1/2}\left(1-\frac{n_l}{2S}\right)^{1/2}\,\psi_{n_l+1} \end{aligned}\right\} \qquad (1.6)$$

$$\hat{n}_l \psi_{n_l} = n_l \psi_{n_l}$$

and satisfy the commutation relations:

$$[S_l^z, S_m^{\pm}] = \pm S_l^{\pm} \delta_{lm}$$
$$[S_l^+, S_m^-] = 2S_l^z \delta_{lm}, \qquad (1.7)$$

where δ_{lm} is the Kronecker δ - function.

In handling problems concerning spin wave interactions, it is expedient to work in the second quantisation formalism i.e. the number operator formalism. This leads to the concept of quasi-particles associated with the spin waves. Let us denote by a_l^+ and a_l, the creation and the annihilation operators respectively which create and destroy spin deviation at site l. These operators are defined by

$$a_l^+ \psi_{n_l} = (n_l+1)^{1/2} \psi_{n_l+1}$$
$$a_l \psi_{n_l} = (n_l)^{1/2} \psi_{n_l-1} \qquad (1.8)$$

Comparing (1.6) and (1.8) we have

$$S_l^+ = (2S)^{1/2} \left(1 - \frac{a_l^+ a_l}{2S}\right)^{1/2} a_l$$
$$S^- = (2S)^{1/2} a_l^+ \left(1 - \frac{a_l^+ a_l}{2S}\right)^{1/2} \qquad (1.9)$$
$$\hat{n}_l = a_l^+ a_l = S - S_l^z$$

The commutation relations for these operators are:

$$[\, a_l, a_m^+ \,] = \delta_{lm} \tag{1.10}$$

The exchange Hamiltonian for the ferromagnet given by (1.1) can thus be written in terms of the new operators as

$$H_{ex} = - \sum_{i,j} J_{ij} \, \underline{S}_i \cdot \underline{S}_j$$

$$= - \sum_{i,j} J_{ij} \left\{ S_i^z . S_j^z + \frac{1}{2} (S_i^+ S_j^- + S_i^- S_j^+) \right\}$$

and assuming that $a_i^+ a_i < 2S$, we can expand the brackets in terms of the equation (1.9). Thus

$$H_{ex} = \text{Const.} + \sum_{i,j} J_{ij} \left\{ S(a_i^+ a_i + a_j^+ a_j - a_i a_j^+ - a_i^+ a_j) \right\}$$

$$+ \text{ higher order terms} \tag{1.11}$$

Now, a spin deviation or disturbance will not remain localised at a particular site $\underline{R}_i$, but will move throughout the crystal, like a wave due to the strong exchange forces.[9] If there is only one spin wave present, it would be an exact eigenstate of the Hamiltonian. However, if there are more than one spin waves present in the lattice, interaction between them will come into play and the Hamiltonian will no longer be diagonal. The

Hamiltonian in this case is split up into two parts - a) quadratic and b) the other containing higher order terms of the operators a and a^+.

Let us introduce Fourier transforms in the reciprocal space defined as

$$\left.\begin{aligned} a_i &= \frac{1}{\sqrt{N}} \sum_{\underline{\lambda}} e^{i\underline{\lambda}.R_i}\, a_{\underline{\lambda}} \\ a_i^+ &= \frac{1}{\sqrt{N}} \sum_{\underline{\lambda}} e^{-i\underline{\lambda}R_i}\, a^*_{\underline{\lambda}} \quad , \end{aligned}\right\} \qquad (1.12)$$

where N is the number of sites in the crystal and $\underline{\lambda}$ the wave vector of the quasi-particle (called Magnon). By using the periodic boundary condition

$$\sum_i e^{i\underline{\lambda}R_i} = N\,\Delta(\underline{\lambda}) \qquad (1.13)$$

where $\Delta(\underline{\lambda}) = 0$ for $\underline{\lambda} \neq 0$ and $\Delta(\underline{\lambda}) = 1$ for $\underline{\lambda} = 0$, we obtain the inverse transformations

$$\left.\begin{aligned} a_{\underline{\lambda}} &= \frac{1}{\sqrt{N}} \sum_i e^{-i\underline{\lambda}.\underline{R}_i}\, a_i \\ a^+_{\underline{\lambda}} &= \frac{1}{\sqrt{N}} \sum_i e^{i\underline{\lambda}.\underline{R}_i}\, a_i^+ \end{aligned}\right\} \qquad (1.14)$$

The operators $a_{\underline{\lambda}}$, $a^+_{\underline{\lambda}}$ satisfy the commutation rules which can be found from (1.10).

$$[\, a_{\underline{\lambda}} , a^{+}_{\underline{\lambda}'} \,] = \delta_{\underline{\lambda}\underline{\lambda}'} \tag{1.15}$$

all other commutators being zero.

From these commutation relations it is evident that Magnons are bosons. In terms of these operators, the diagonal part of the Hamiltonian takes the form

$$H_o = \sum \hbar w_{\underline{\lambda}} \left(a^{+}_{\underline{\lambda}} a_{\underline{\lambda}} + \frac{1}{2} \right) , \tag{1.16}$$

where $\hbar w_\lambda$ is the energy of the magnon with the wave vector $\underline{\lambda}$ and takes different expressions for different systems.

For a ferromagnet, the dispersion relation i.e. the dependence of w_λ on $\underline{\lambda}$ takes a simple form in the long wave length limit namely $\underline{\lambda} . R_i \ll 1$:

$$\hbar w_\lambda = 2JSZ\,(1-\gamma_{\underline{\lambda}}) = 2JSZ\, a^2 \lambda^2 \tag{1.17}$$

for a cubic crystal where J is the exchange integral for nearest neighbours and z their number and

$$\gamma_{\underline{\lambda}} = \frac{1}{z} \sum_h e^{i\underline{\lambda}\,\underline{R}_h} ; \tag{1.18}$$

$\underline{R}_h$ is the nearest neighbour distance.

C. ANTIFERROMAGNETIC MAGNONS

Let two interpenetrating cubic lattices of a simple anti-ferromagnet together form a body centred cubic lattice. The spin of one sub-lattice points up and that of the second points down. In an external field the spins line up antiferromagnetically in a plane perpendicular to the applied field. The degeneracy associated with the orientation in the same plane would remain. The degeneracy is removed by invoking an anisotropy field[5] in the + z direction so that the spins of the sub-lattice 1 are aligned in z-direction and those of the sub-lattice 2 in -z direction.

The Hamiltonian of the system is expressed as

$$H_{AF} = H_{ex} + H_z + H_{am}$$

$$= 2J \sum_{l,m} S_l \cdot S_m - Hg\mu_B \sum_{j=l,m} S_J^z - H_A g u_B \sum_{l,m} (S_l^z - S_m^z), \qquad \text{........(1.19)}$$

where l spans the sublattice 1 and m spans the sublattice 2; H_A is the anisotropy field and J is the exchange constant. The magnitude of the spin quantum number at each sub-lattice is the same.

The spin deviation operators for the two sub-lattices have to be defined somewhat differently. These are:

$$\left.\begin{aligned} S_l^+ &= (2S)^{1/2} \left(1 - \frac{n_l}{2S} \right)^{1/2} a_l \\ S_l^- &= (2S)^{1/2} a_l^+ \left(1- \frac{n_l}{2S} \right)^{1/2} \\ S - S_l^z &= a_l^+ a_l = n_l \end{aligned}\right\} \qquad (1.20)$$

Likewise for the other sub-lattice

$$\left.\begin{aligned} S_m^+ &= (2S)^{1/2} d_m^+ \left(1 - \frac{n_m}{2S} \right)^{1/2} \\ S_m^- &= (2S)^{1/2} \left(1 - \frac{n_m}{2S} \right)^{1/2} d_m \\ S + S_m^z &= d_m^+ d_m = n_m \end{aligned}\right\} \qquad (1.21)$$

These operators also satisfy the commutation relations

$$\left.\begin{aligned} [\, a_l, a_{l'}^+ \,] &= \delta_{ll'} \\ [\, d_m, d_{m'}^+ \,] &= \delta_{mm'} \end{aligned}\right\}, \qquad (1.22)$$

the other commutation brackets being zero. Neglecting the terms involving interactions among magnons, the Hamiltonian can be transformed to

$$H_{AF} = \text{constant} + 2JS \sum_{l,m} (a_l d_m + a_l^+ d_m^+ + a_l^+ a_l + d_m^+ d_m)$$

$$+ g\mu_B \left\{ H \Big(\sum_l a_l^+ a_l - \sum_m d_m^+ d_m \Big) + H_A \Big(\sum_l a_l^+ a_l + \sum_m d_m^+ d_m \Big) \right\}$$

.... (1.23)

As before we use the spin wave fourier transformations on the spin deviation operators, which are given by

$$\left.\begin{aligned} a_l &= \left(\frac{2}{N}\right)^{1/2} \sum_{\underline{\lambda}} e^{-i\underline{\lambda}\underline{R}_l}\, a_{\underline{\lambda}} \\ a_l^+ &= \left(\frac{2}{N}\right)^{1/2} \sum_{\underline{\lambda}} e^{i\underline{\lambda}\underline{R}_l}\, a^+_{\underline{\lambda}} \\ d_m &= \left(\frac{2}{N}\right)^{1/2} \sum_{\underline{\lambda}} e^{-i\underline{\lambda}\underline{R}_m}\, d_{\underline{\lambda}} \\ d_m^+ &= \left(\frac{2}{N}\right)^{1/2} \sum_{\underline{\lambda}} e^{i\underline{\lambda}\underline{R}_m}\, d^+_{\underline{\lambda}} \end{aligned}\right\} \qquad (1.24)$$

where $a_{\underline{\lambda}}$ and $d_{\underline{\lambda}}$ are the spin wave operators. Here the propagation vectors $\underline{\lambda}$ span the N/2 points of the first Brillouin Zone on the reciprocal space of the lattice.

Thus we get

$$H_{AF} = 2JS \sum_{\underline{\lambda}} \gamma_{\underline{\lambda}} \left(a_{\underline{\lambda}} d_{\underline{\lambda}} + a^+_{\underline{\lambda}} d^+_{\underline{\lambda}} + a^+_{\underline{\lambda}} a_{\underline{\lambda}} + d^+_{\underline{\lambda}} d_{\underline{\lambda}}\right)$$

$$+ gu_B(H+H_A) \sum_{\underline{\lambda}} a^+_{\underline{\lambda}} a_{\underline{\lambda}} + (H_A - H)\, d^+_{\underline{\lambda}} d_{\underline{\lambda}} \qquad (1.25)$$

where $\gamma = 1/z \sum_{h} e^{i.\underline{\lambda}.\underline{R}_h}$, $\underline{R}_{\underline{h}} = \underline{R}_{\underline{l}} - \underline{R}_{\underline{m}}$ (1.26)

The summation over h extends to the nearest neighbour interactions. The Hamiltonian (1.25) is not diagonal.

To achieve this we introduce the new creation and annihilation operators (α_{λ}^{+} , α_{λ}) and (β_{λ}^{+}, β_{λ}) given by

$$\left.\begin{aligned} a_{\lambda} &= \alpha_{\lambda} \cosh \Theta_{\lambda} + \beta_{\lambda}^{+} \sinh \Theta_{\lambda} \\ a_{\lambda}^{+} &= \alpha_{\lambda}^{+} \cosh \Theta_{\lambda} + \beta_{\lambda} \sinh \Theta_{\lambda} \\ d_{\lambda} &= \alpha_{\lambda}^{+} \sinh \Theta_{\lambda} + \beta_{\lambda} \cosh \Theta_{\lambda} \\ d_{\lambda}^{+} &= \alpha_{\lambda} \sinh \Theta_{\lambda} + \beta_{\lambda}^{+} \cosh \Theta_{\lambda} \end{aligned}\right\} \qquad (1.27)$$

In order that the Hamiltonian be in the diagonal form and the new operators satisfy the Boson commutation relations, the parameter Θ must satisfy the relation

$$\tanh 2\Theta_{\lambda} = - \left(\frac{w_e \gamma_{\lambda}}{w_e + w_A} \right) , \qquad (1.28)$$

where

$$\left.\begin{aligned} w_e &= 2zSJ/\hbar \\ w_A &= g\mu_B H_A / \hbar \end{aligned}\right\} \qquad (1.29)$$

$$H_{AF(Magnon)} = \sum_{\lambda} \hbar w_{\lambda}^{+} (\alpha_{\lambda}^{+}\alpha_{\lambda} + \tfrac{1}{2}) + \sum_{\lambda} \hbar w_{\lambda}^{-} (\beta_{\lambda}^{+}\beta_{\lambda} + \tfrac{1}{2}) \quad \ldots \quad (1.30)$$

with

$$\left.\begin{aligned} w^{\pm} &= [(w_e + w_A)^2 - w_e^2 \gamma_{\lambda}^2] \pm w_H \\ \text{and} \quad w_H &= g\mu_B H/\hbar \end{aligned}\right\} \qquad (1.31)$$

In the long wavelength limit, we find the dispersion relation for antiferromagnet to be linear in wave-vector as distinguished from the parabolic relationship in the case of ferromagnets.

D. PHONONS

The spin system at finite temperatures can be described as an assembly of spin waves and the elementary excitations of such a system are described as quasi-particles called magnons. In a crystal, there exists another elementary excitation namely the lattice waves. The ions in a crystal are never quiescent, they execute small oscillations about their equilibrium positions. But since an ion is strongly coupled to its neighbouring ions by elastic and other interionic forces, the small oscillation spreads in the form of a disturbance through the crystal. This leads to a collective motion of the ions which gives rise to very important thermodynamic effects and produces interactions with other entities such as electrons and spin waves.

Thus when considering all the interactions, we should consider the total Hamiltonian as

$$H = H_L + H_{el} \qquad (1.32)$$

where H_L is the lattice Hamiltonian and H_{el} is the electron Hamiltonian including the two body interactions of all types. Let us denote by $\underline{P}_{1,b}$ and $\underline{h}_{1,b}$ the momentum and the displacement of the b^{th} ion in the 1^{th} unit cell (here $\underline{1}$ also denotes the vector to the unit cell from a fixed origin and $\underline{b}$ the vector to the ion from a fixed ion in the cell). Thus H_L is explicitly given by

$$H_L = \frac{1}{2} \sum_{1,b} \left(\frac{1}{m_b} \right) \underline{P}_{1,b} \, \underline{P}_{1,b} + V \qquad (1.33)$$

where m_b is the mass of the b^{th} ion in the unit cell. The two terms in (1.33) represent the kinetic and the potential energies of the ions respectively.

Expanding the potential energy in Taylor series, we have,

$$V = V_o + \frac{1}{2} \sum_{1,b} \sum_{1'b'} \underline{h}_{1,b} \left[\frac{\partial^2 V}{\partial \underline{h}_{1,b} \, \partial \underline{h}_{1'b'}} \right] \underline{h}_{1',b'} + \cdots \qquad \ldots\ldots\ldots (1.34)$$

Substituting this in (1.33) we have

$$H_L = \frac{1}{2} \sum_{1,b} \left(\frac{1}{m_b} \right) P_{1,b} \cdot P_{1,b}$$

$$+ \frac{1}{2} \sum_{1,b} \sum_{1'b'} \underline{h}_{1,b} \left[\frac{\partial^2 V}{\partial h_{1,b} \, \partial h_{1'b'}} \right] h_{1',b'} + \cdots \qquad \ldots\ldots\ldots (1.35)$$

Here it is convenient to introduce second quantisation by defining[10]

$$\left.\begin{aligned} \underline{H}_{l,b} &= -\,i\left(\frac{\hbar}{2Nm_b}\right) \sum_{\underline{q}} \underline{e}_{\underline{q}b}\, w_{\underline{q}}^{-1/2} (b_{\underline{q}}^{+} - b_{-\underline{q}})\, e^{-i\underline{q}\underline{l}} \\ \underline{P}_{l,b} &= \left(\frac{\hbar m_b}{2N}\right)^{1/2} \sum \underline{e}_{\underline{q}b}\, w_{\underline{q}}^{1/2} (\underline{b}_{q} + \underline{b}_{-q}^{+})\, e^{i\underline{q}\underline{l}}, \end{aligned}\right\} \quad (1.36)$$

where $\underline{q}$ represents the wave vector and the branch index of the lattice wave, $\underline{e}_{\underline{q}b}$ the po arisation vector and $w_{\underline{q}}$ the mode branch frequency of the lattice wave. In terms of the new operators, the lattice Hamiltonian becomes

$$H_L = \sum_{q} \hbar w_{\underline{q}} (b_{\underline{q}}^{+} b_{\underline{q}} + \tfrac{1}{2}) + \text{higher order terms} \quad (1.37)$$

with $N_q = b_q^{+} b_q$, the occupation number operator which has only positive integers as its eigenvalues.

From the analogy of 1.37 with 1.16 and 1.30 we can interpret the lattice vibrational field as consisting of a system of non-interacting particles called phonons, each having an energy $\hbar w_{\underline{q}}$. The operator $b_{\underline{q}}^{+}$ creates a phonon of wave vector $\underline{q}$ in a given branch and $b_{\underline{q}}$ destroys such a phonon.

The eigenfunctions of the Hamiltonian (1.37) can be denoted in the occupation number representation by $| \ldots N_q \ldots \rangle$ and the above operators have then the properties

$$\left.\begin{aligned} b_{\underline{q}}^{+} \left| \ldots . N_q \right\rangle &= (N_q+1)^{1/2} \left| \ldots N_q+1 \ldots \right\rangle \\ b_{\underline{q}} \left| \ldots . N_q \right\rangle &= (N_q)^{1/2} \left| \ldots N_q-1 \ldots \right\rangle \end{aligned}\right\} \quad (1.38)$$

They also satisfy the Boson commutation relation

$$[\, b_{\underline{q}},\, b_{\underline{q}'}^{+} \,] = \delta_{\underline{q}\underline{q}'} \qquad (1.39)$$

E. PHONON-MAGNON INTERACTION

A precise knowledge of the energy spectrum of excitations is essential in the study of the properties of a magnetically ordered system. While the dispersion relations of pure phonon and magnon modes are already known, the modification introduced by their mutual interactions has not been the subject of detailed study. Various workers[11-13] have studied the various mechanisms of magnon-phonon interaction and their effect on the transport and relaxation phenomena has been delineated.

However, there are experimental indications that magnon-phonon interaction terms would influence the energy spectrum of magnon and phonon modes. In fact, in the famous quadratic dispersion relations of Acoustic magnons, relation of energy $E_{\underline{\lambda}}$ with its propagation vector $\underline{\lambda}$ namely $E_{\underline{\lambda}} = D(T)\,\lambda^2$, the coefficient D(T) is a function of temperature. This temperature dependence cannot be explained on the harmonic model and one has to invoke renormalisation of these modes according to their interaction with other modes. Likewise, phonon modes are also expected to be influenced by such interactions in the form of changes in dispersion relations or the Debye temperature.

F. THE GREEN'S FUNCTION METHOD

The spin-wave concept has proved to be of great utility in describing the behaviour of magnetically ordered systems. The low temperature excitations are very well explained on this concept and expressions for magnetisation derived from it seem to be in accord with experimental evidence. However, to obtain a more complete theory, Green's function method initially developed for problems in field theory, has been applied to statistical mechanics and magnetism.

The method is simple in its formulation and interpretation. When combined with the spectral representation, it becomes a powerful tool for attacking various types of problems. The first application of the Green's function method to non-relativistic solid state theory was made by Bonch Bruevich[14] in 1955 and therefore it has been used by various workers in connection with a wide variety of statistical problems. Matsubara[15] tried to formulate a method to suit finite temperatures but his Green's functions were time independent. A complete generalisation of this method was achieved by several Russian workers, cited in references.[16-19]

Let A(x) and B(x') be any two operators in the Heisenberg representation, where x contains the spatial coordinates $\underline{x}$ and the time co-ordinate t. We can define the Green's functions as follows:[20,21,22]

$$G_r(x,x') \equiv \langle\langle A(x);B(x')\rangle\rangle_r \equiv -i\theta(t-t')\langle [A(x),B(x')]_\eta \rangle$$

$$\equiv -i\theta(t-t')\langle A(x)B(x')\rangle + i\,\theta(t-t')\langle B(x')A(x)\rangle \qquad \ldots\ldots\ldots (1.40)$$

$$G_a(x,x') \equiv \langle\langle A(x);B(x')\rangle\rangle_a \equiv i\theta(t'-t)\langle [A(x),B(x')]_\eta \rangle$$

$$\equiv i\theta(t'-t)\langle A(x)B(x')\rangle - i\eta\theta(t'-t)\langle B(x')A(x)\rangle \qquad \ldots\ldots\ldots (1.41)$$

$$G_c(x,x') = \langle\langle A(x);B(x')\rangle\rangle_c = -i\langle TA(x)B(x')\rangle$$

$$= -i\theta(t-t')\langle A(x)B(x')\rangle - i\eta\theta(t'-t)\langle B(x')A(x)\rangle \qquad \ldots\ldots (1.42)$$

where η is a disposable constant and is taken to be +1 or -1 according to A and B are Fermi or Bose operators. Generally speaking, A and B are neither Fermion nor Boson operators, for products of operators can satisfy complicated commutation relations.

The sign of η for multiple time Green's functions is uniquely determined depending on whether an odd or even permutation of parity 'p' for the Fermi operators in these functions is invo ved in going over to the chronological order i.e. $\eta = (-1)^p$.

T is Dyson's chronological operator and arranges Heisenberg operators occuring in a given product from right to left in the increasing order of their time arguments and multiplies the chronological products thus obtained by $\eta = (-1)^p$, where p is the parity of the permutation of the Fermion operator when we change f om the given order to chronological order i.e.

$$T(A_1(x_1), \ldots A_n(x_n)) = \eta A_{j_1}(x_{j_1}) \ldots A_{j_n}(x_{j_n}),$$

$$t_{j_1} > t_{j_2} > \quad t_{j_n} .$$

Here $\Theta(t)$ is the Heaviside step function defined by

$$\Theta(t) = 0 \quad \text{if} \quad t < 0$$
$$\text{and } 1 \quad \text{if} \quad t > 0 .$$

For any operator P, $\langle P \rangle$ denotes the average over a grand cannonical assemble

$$\langle P \rangle = \frac{Tr\,(Pe^{-\mathcal{H}/k_BT})}{Tr\,(e^{-\mathcal{H}/k_BT})} = Z^{-1}\, Tr\,(Pe^{-\beta\mathcal{H}}) \,, \qquad (1.43)$$

The operator $\mathcal{H}$ includes a term with the chemical potential μ; $\mathcal{H} = H - \mu N$, where N is the number operator. For interactions in which the creation and the annihilation operators always occur in pair, so that the number operator N is a q-number, there is no essential distinction between $\mathcal{H}$ and H. Of course, for Bosons $\mu = 0$ and again $\mathcal{H} = H$.

Z is the grand partition function, H is the Hamiltonian, k_B the Boltzmann's constant and T the temperature. The different Green's functions G_r, G_a and G_c are known as retarded, advanced and causal Green's functions respectively. The function of physical interest is the retarded commutator type Green's function G_r and we shall denote it only by G.

Let us introduce the spectral function $J_{BA}(E)$ which is Fourier transform of the correlation function of two operators A(t) and B(o) by

$$\langle B(o)A(t)\rangle \, e^{-\alpha|t|} = \int_{-\infty}^{+\infty} J_{BA}(E)\, e^{-iEt/\hbar}\, dE$$

$$= Z^{-1}\, \mathrm{Tr}\, (e^{-\beta H}\, B e^{iHt/\hbar} A e^{-iHt/\hbar})\, e^{-\alpha t}, \qquad \text{........ (1.44)}$$

where we have introduced an infinitesimal convergence factor $e^{-\alpha|t|}$ in order to ensure the convergence of the Fourier transform. In Heisenberg representation,

$$A(t) = e^{iHt/\hbar} A e^{-iHt/\hbar} \qquad (1.45)$$

The inverse transform of Eqn (1.44) is

$$J_{BA}(E) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \langle B(o)A(t)\rangle \, e^{iEt/\hbar - \alpha|t|} \, dt \qquad (1.46)$$

Now, we change the order of operation as follows:

$$\langle B(o)A(t)\rangle = z^{-1} \, Tr \, (e^{-\beta H} B e^{iHt/\hbar} A e^{-iHt/\hbar})$$

$$= z^{-1} \, Tr \, (e^{iHt/\hbar} A e^{-iHt/\hbar} e^{-\beta H} B)$$

$$= z^{-1} \, Tr \, (e^{-\beta H} e^{+iH(t/\hbar - i\beta)} A e^{-iH(t/\hbar - i\beta)} B)$$

$$= \langle A(t/\hbar - i\beta) \, B(o)\rangle$$

Then

$$J_{BA}(E) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \langle A(t/h - i\beta) \, B(o) \rangle \, e^{iEt/\hbar - \alpha|t|} dt$$

Distorting the path of integration in the complex t-plane by the analyticity condition, we obtain

$$J_{BA}(E) = \frac{e^{-\beta E}}{2\pi\hbar} \int_{-\infty + i\beta}^{\infty + i\beta} \langle A(s)B(o)\rangle \, e^{iEs/\hbar - \alpha|s|} \, ds$$

$$= \frac{e^{-\beta E}}{2\pi\hbar} \int_{-\infty}^{\infty} \langle A(t)B(o)\rangle \, e^{iEt/\hbar - \alpha|t|} \, dt$$

Hence

$$e^{\beta E} J_{BA}(E) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \langle A(t)B(o)\rangle \; e^{iEt/\hbar-\alpha|t|}\, dt \qquad (1.47)$$

Combining equations (1.46) and (1.47) we have

$$(e^{\beta E} \pm 1) J_{BA}(E) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \langle [A(t), B(o)] \rangle \; e^{iEt/\hbar-\alpha|t|}\, dt \qquad \ldots\ldots (1.48)$$

Since t varies from $-\infty$ to ∞ the integral splits up into two parts in the following form

$$(e^{\beta E} \pm 1) J_{BA}(E) = \frac{1}{2\pi i\hbar}\left[\int_{-\infty}^{\infty} \langle\langle A(t);B(o)\rangle\rangle_r \; e^{i(E/\hbar+i\alpha)t}\, dt - \int_{-\infty}^{\infty} \langle\langle A(t);B(o)\rangle\rangle_a \; e^{i(E/\hbar-i\alpha)t}\, dt \right]$$

$$= \frac{1}{i} \langle\langle A; B\rangle\rangle_{E+i\alpha} - \langle\langle A;B\rangle\rangle_{E-i\alpha} , \qquad \ldots\ldots (1.49)$$

where the Fourier transforms are defined by

$$\left.\begin{aligned} \langle\langle A;B\rangle\rangle_{E+i\alpha} &= \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \langle\langle A(t);B(o)\rangle\rangle_r \; e^{i(E+i\alpha)t}\, dt \\ \langle\langle A;B\rangle\rangle_{E-i\alpha} &= \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \langle\langle A(t);B(o)\rangle\rangle_a \; e^{i(E-i\alpha)t}\, dt \end{aligned}\right\} \qquad (1.50)$$

The relation (1.49) is very important for the calculation of many properties. The equation of Motion for the Green's functions are:

$$i\hbar\frac{d}{dt}\langle\langle A(t);B(o)\rangle\rangle_{r,a} = \hbar\,\delta(t)\langle[A,B]_{\mp}\rangle + \langle\langle[A(t),H];B(o)\rangle\rangle_{r,a},$$

where $\delta(t)$ is Dirac's Delta function.

The fourier transform of this equation yields

$$E\,\langle\langle A;B\rangle\rangle_E = -\frac{1}{2\pi}\langle[A,B]_{\mp}\rangle + \langle\langle[A,H]_- ;\ B\rangle\rangle_E \qquad (1.51)$$

Since the spectral functions are related to $\langle\langle A;\ B\rangle\rangle_{E\mp i\alpha}$, it is required to know only the Fourier transform of the Green's function. It may be pointed out that the Fourier transformed equation of motion is the same whether we start from the advanced or retarded Green's function.

In the present work, the effect of Magnon-Phonon interaction on the renormalisation of the quasi-particle energies is studied. The Hamiltonians for the Ferro-magnetic and Antiferromagnetic systems are formulated in the second quantisation representation. Then we introduce double time temperature Green's functions for magnons and phonons. A set of coupled equations of motion are derived and solved after suitable decoupling approximations. Also, the life-time of each quasi-particle is derived from the relation between relaxation frequency and the imaginary part of self-energy.

CHAPTER - 2

FERROMAGNETS

Let us consider a ferromagnetic system crystallising in a body-centred cubic structure fo convenience. The treatment, is, however applicable to any general exchange coupled ferromagnetic system. The Hamiltonian in the spin operator representation is given by

$$H = \text{constant} + H_L - \sum_{l<m} 2 J_{eff}(R_{lm})\, \underline{S}_l \cdot \underline{S}_m$$

$$- g \mu_B \underline{H} \sum_l S_l^z$$

$$+ 8 \sum_{l \neq m} \left[\sum_h {}^e\underline{J}(R_{lm}) \cdot \delta \underline{R}_h \right] (\underline{S}_l \cdot \underline{S}_m) \qquad (2.1)$$

where H_L is the lattice Hamiltonian, the third and the fourth terms are isotropic exchange and zeeman energies and the last term represents the interaction between the exchange coupled spins and the lattice. We have neglected the anisotropy energy and dipole-dipole interaction terms. $\underline{S}_l$, $\underline{S}_m$ in the above equation are the spin operators for the spins at sites l and m respectively. J_{eff} is the isotropic exchange integrals, $\underline{H}$ the external magnetic field along the z-direction and $\delta \underline{R}_h$ is the relative displacement of the two atoms at $\underline{l}$ and $\underline{m}$. ${}^e\underline{J}(\underline{R}_{lm})$ represents the modification of the effective exchange integral owing to crystal field oscillations and is given by

$$ {}^{e}J(\underline{R}_{lm}) = \sum_{\underline{\lambda}} {}^{\lambda}J_{eff}(R_{lm}) \langle \phi_{\lambda} | \underline{V}^{h} | \phi_{l} \rangle / \Delta E_{l\lambda} \qquad (2.2) $$

${}^{\lambda}J_{eff}$ refers to those effective exchange integrals which involve one excited orbital, which connects it with $\langle \phi_{\lambda} | V_{h} | \phi_{l} \rangle$. $\langle \phi_{\lambda} | V^{h} | \phi_{l} \rangle$, is the matrix element of the operator $\underline{V}^{h}$ ($= \partial \underline{V} / \partial \underline{R}_{h}$, V being the crystal field) connecting the orbital states ϕ_{l} and ϕ_{λ}; $\Delta E_{l\lambda}$ is the energy denominator.

In order to go over to the second quantised representation, we make use of the following transformations

$$ \delta R_{h} = \frac{1}{\sqrt{N}} \sum_{\underline{q}} \underline{g}_{\underline{q}} (b^{+}_{\underline{q}} - b_{-\underline{q}}) \left(e^{i\underline{q}.\underline{R}^{o}_{l}} - e^{i\underline{q}.\underline{R}^{o}_{m}} \right) \qquad (2.3) $$

$$ g_{q} = (-i) \; \underline{\varepsilon}_{\underline{q}} (\hbar / 2w_{q} M)^{1/2} \; , \qquad (2.4) $$

where $b^{+}_{\underline{q}}$ and $b_{\underline{q}}$ are the phonon creation and annihilation operators respectively corresponding to the wave-vector $\underline{q}$, $\underline{\varepsilon}_{\underline{q}}$ is the polarisation vector, w_{q} the mode-branch frequency, M the atomic mass.

The spin operators are related to the spin wave operators by the following transformation:

$$\left.\begin{aligned} S_l^{+} &= \sqrt{\frac{2S}{N}} \sum_{\underline{\lambda}} e^{i\underline{\lambda}.\underline{R}_l} a_{\underline{\lambda}} \\ S_l^{-} &= \sqrt{\frac{2S}{N}} \sum_{\underline{\lambda}} e^{-i\underline{\lambda}.\underline{R}_l} a_{\underline{\lambda}}^{+} \\ S_l^{z} &= S - \frac{1}{N} \sum_{\underline{\lambda},\underline{\lambda}'} e^{-i(\underline{\lambda}-\underline{\lambda}')\underline{R}_l} a_{\underline{\lambda}}^{+} a'_{\underline{\lambda}} \end{aligned}\right\} \qquad (2.5)$$

Making use of these transformations, the Hamiltonian (2.1) can be recast into the form

$$H = H_m + H_p + H_{pm} \quad , \qquad (2.6)$$

where

$$H_m = \sum_{\underline{\lambda}} \hbar w_{\underline{\lambda}} \left(a_{\underline{\lambda}}^{+} a_{\underline{\lambda}} + \frac{1}{2}\right) \qquad (2.7)$$

$$H_p = \sum_{\underline{q}} \hbar w_{\underline{q}} \left(b_{\underline{q}}^{+} b_{\underline{q}} + \frac{1}{2}\right) \qquad (2.8)$$

$$H_{pm} = \sum_{\underline{\lambda},\underline{q}} \phi_{\underline{\lambda}\underline{q}} \, a_{\underline{\lambda}-\underline{q}}^{+} a_{\underline{\lambda}} b_{\underline{q}}^{+} - \text{hermitian conjugate} \qquad (2.9)$$

with

$$\hbar w_{\underline{\lambda}} = 2JSz\,(1-\gamma_{\underline{\lambda}}) + g\,\mu_B H \qquad (2.10)$$

$$\gamma_{\underline{\lambda}} = \frac{1}{z} \sum_{h} e^{i\underline{\lambda}.\underline{R}_h} \qquad (2.11)$$

$\phi_{\underline{\lambda}\underline{q}}$ is the magnon-phonon coupling coefficient and is given by

$$\phi_{\lambda q} = \frac{8S}{\sqrt{N}} ({}^{e}J_{eff}(R_h) \cdot g_q)[\gamma_\lambda - \gamma_{\lambda-q} + \gamma_q - 1] \quad (2.12)$$

In order to study the effect of phonon-magnon interactions on the energy spectrum of collective modes in the case of ferromagnet, we start with the single particle magnon and phonon Green's functions:

$$\langle\langle a_\lambda(t); a^+_\lambda(t') \rangle\rangle \quad \text{and} \quad \langle\langle b_q(t); b^+_q(t') \rangle\rangle$$

Taking first the case of magnon Green's function and noting that magnons are bosons, the equation of motion is

$$i\hbar \frac{d}{dt} \langle\langle a_\lambda(t); a^+_\lambda(t') \rangle\rangle = \hbar\delta(t-t') \langle [a_\lambda, a^+_\lambda]_{-1} \rangle$$

$$+ \langle\langle [a_\lambda(t); H(t)]; a^+_\lambda(t') \rangle\rangle \quad (2.13)$$

Now considering the total Hamiltonian $H_{ferro} = H_p + H_m + H_{mp}$ as determined in equations (2.7), (2.8) and (2.9) and noting that various commutators are as under:

$$[a_\lambda, a^+_\lambda] = 1 \quad (2.14)$$

$$[a_\lambda, H_p] = 0 \quad (2.15)$$

$$[a_\lambda, H_m] = A_\lambda a_\lambda \quad (2.16)$$

$$[a_\lambda, H_{mp}] = \sum_q \phi_{(\lambda+q)q} a_{\lambda+q} (b^+_q - b_{-q}) \quad (2.17)$$

The equation of motion becomes:

$$i\hbar \frac{d}{dt} \langle\langle a_{\lambda}(t);\ a^{+}_{\lambda}(t')\rangle\rangle = \hbar\delta(t-t') + A_{\lambda}\langle\langle a_{\lambda}(t);\ a^{+}_{\lambda}(t')\rangle\rangle$$

$$+ \sum_{q} \phi_{(\lambda+q),q} \langle\langle a_{\lambda+q}(t)(b^{+}_{q}(t)-b_{-q}(t);a^{+}_{\lambda}(t')\rangle\rangle \qquad \text{.......} \ (2.18)$$

Likewise, the equation of motion for phonon Green's function will be

$$i\hbar \frac{d}{dt} \langle\langle b_{q}(t);\ b^{+}_{q}(t')\rangle\rangle$$

$$= \hbar\delta(t-t')\langle [b_{q}, b^{+}_{q}] \rangle + \langle\langle [b_{q}(t),\ H_{ferro}(t)];b^{+}_{q}(t')\rangle\rangle \qquad \text{......} \ (2.19)$$

Noting that the various commutators are:

$$[b_{q},\ b^{+}_{q}] = 1 \qquad (2.20)$$

$$[b_{q},\ H_{m}] = 0 \qquad (2.21)$$

$$[b_{q},\ H_{p}] = \hbar w_{q}.b_{q} \qquad (2.22)$$

$$[b_{q},\ H_{mp}] = \sum_{\lambda} \phi_{\lambda q}\ a^{+}_{\lambda-q}\ a_{\lambda} \qquad (2.23)$$

The equation of motion for phonon becomes

$$i\hbar \frac{d}{dt} \langle\langle b_q(t); b_q^+(t') \rangle\rangle =$$

$$= \hbar\delta(t-t') + \hbar w_q \langle\langle b_q(t); b_q^+(t') \rangle\rangle$$

$$+ \sum_{\lambda} \phi_{\lambda q} \langle\langle a_{\lambda-q}^+(t)\, a_\lambda(t); b_q^+(t') \rangle\rangle \tag{2.24}$$

In order to solve the coupled equations in (2.18) and (2.24) we require the equations of motion of the higher order Green's functions on the right hand side of these equations.

The various commutators required for the purpose are:

$$[a_{\lambda+q} b_q^+; a_\lambda^+] = b_q^+ \delta_{\lambda+q,\lambda} \tag{2.25}$$

$$[a_{\lambda+q} b_q^+, H_p] = -\hbar w_q\, a_{\lambda+q}\, b_q^+ \tag{2.26}$$

$$[a_{\lambda+q} b_q^+, H_m] = A_{\lambda+q}\, a_{\lambda+q}\, b_q^+ \tag{2.27}$$

$$[a_{\lambda+q} b_q^+, H_{mp}] = \sum_{q_1} \phi_{(\lambda+q+q_1)q_1}\, a_{\lambda+q+q_1}\, b_{q_1}^+ . b_q^+$$

$$+ \phi_{(\lambda),(-q)}\, a_\lambda + \sum_{q_1} \phi_{(\lambda+q+q_1),q_1}\, a_{\lambda+q+q_1}\, b_q^+ \underline{b}_{q_1}$$

$$- \sum_{\lambda_1} \phi_{(\lambda_1,-q)}\, a_{\lambda+q}\, a_{\lambda_1+q}^+\, a_{\lambda_1} \tag{2.28}$$

$$[\, a_{\lambda+q}\, b_{-q},\, a^{+}_{\lambda}\,] = b_{-q} \cdot \delta_{\lambda+q,\lambda} \tag{2.29}$$

$$[\, a_{\lambda+q}\, b_{-q},\, H_p\,] = \hbar w_q \; a_{\lambda+q}\, b_{-q} \tag{2.30}$$

$$[\, a_{\lambda+q}\, b_{-q},\, H_m\,] = A_{\lambda+q}\, a_{\lambda+q}\, b_{-q} \tag{2.31}$$

$$[\, a_{\lambda+q}\, b_{-q},\, H_{pm}\,] = -\, \phi_{\lambda,(-q)}\, a_{\lambda} + \sum_{q_1} \phi_{(\lambda+q+q_1)q_1}\, a_{\lambda+q+q_1}\, b_{-q}\, b^{+}_{q_1} + \sum_{\lambda_1} \phi_{(\lambda_1-q)}\, a_{\lambda+q}\, a_{\lambda_1+q}\, a_{\lambda_1} + \sum_{q_1} \phi_{(\lambda+q+q_1)q_1}\, a_{\lambda+q+q_1}\, b_{-q}\, b_{-q_1} \tag{2.32}$$

and

$$[\, a^{+}_{\lambda-q}\, a_{\lambda},\, b_q\,] = 0 \tag{2.33}$$

$$[\, a^{+}_{\lambda-q}\, a_{\lambda},\, H_m\,] = (A_{\lambda} - A_{\lambda-q})\, a^{+}_{\lambda-q}\, a_{\lambda} \tag{2.34}$$

$$[\, a^{+}_{\lambda-q}\, a_{\lambda},\, H_p\,] = 0 \tag{2.35}$$

$$[\, a^{+}_{\lambda-q}\, a_{\lambda},\, H_{mp}\,] =$$

$$+ \sum_{q_1} \phi_{(\lambda+q_1)q_1}\, a^{+}_{\lambda-q}\, a_{\lambda+q_1}\, b^{+}_{q_1}$$

$$- \sum_{q_1} \phi_{(\lambda-q),q_1}\, a_{\lambda-q-q_1}\, a_{\lambda}\, b^{+}_{q_1}$$

$$- \sum_{q_1} \phi_{(\lambda-q),q_1}\, a^{+}_{\lambda-q-q_1}\, a_{\lambda}\, b_{-q_1}$$

$$+ \sum_{q_1} \phi_{(\lambda-q)q_1}\, a^{+}_{\lambda-q}\, a_{\lambda+q_1}\, b_{-q_1} \qquad (2.36)$$

The equations of motion of the higher order Green's functions are:

$$i\hbar \frac{d}{dt} \langle\langle a_{\lambda+q}(t) b^{+}_{q}(t);\, a^{+}_{\lambda}(t') \rangle\rangle =$$

$$= \hbar\delta(t-t')\langle b^{+}_{q}\rangle \delta_{\lambda+q,\lambda} + (A_{\lambda+q} - \hbar w_q)\langle\langle a_{\lambda+q}(t) b^{+}_{q}(t);\, b^{+}_{q}(t')\rangle\rangle$$

$$+ \sum_{q_1} \phi_{\lambda+q+q_1,q_1} \langle\langle a_{\lambda+q+q_1}(t)\, b^{+}_{q_1}(t) b^{+}_{q}(t');\, a^{+}_{\lambda}(t')\rangle\rangle$$

$$- \phi_{\lambda(-q)} \langle\langle a_{\lambda}(t);\, a^{+}_{\lambda}(t')\rangle\rangle - \sum_{q_1} \phi_{\lambda+q+q_1,q_1} \langle\langle a_{\lambda+q+q_1}(t)\, b^{+}_{q}(t) b_{-q_1}(t);\, a^{+}_{\lambda}(t')\rangle\rangle$$

$$+ \sum_{\lambda_1} \phi_{\lambda_1(-q)} \langle\langle a_{\lambda+q}(t)\, a^{+}_{\lambda_1+q}(t)\, a_{\lambda_1}(t);\, a^{+}_{\lambda}(t')\rangle\rangle \quad \ldots\ldots \quad (2.37)$$

$$i\hbar \frac{d}{dt} \langle\langle a_{\lambda+q}(t) b_{-q}(t);\ a^{+}_{\lambda}(t') \rangle\rangle$$

$$= \hbar\delta(t-t')\delta_{\lambda+q,\lambda}\langle b_{-q}\rangle + \hbar w_q \langle\langle a_{\lambda+q}(t) b_{-q}(t); a^{+}_{\lambda}(t') \rangle\rangle$$

$$+ A_{\lambda+q}\langle\langle a_{\lambda+q}(t) b_{-q}(t); a^{+}_{\lambda}(t') \rangle\rangle - \phi_{\lambda,(-q)}\langle\langle a_{\lambda}(t);\ a^{+}_{\lambda}(t') \rangle\rangle$$

$$+ \sum_{q_1} \phi_{\lambda+q+q_1,q_1} \langle\langle a_{\lambda+q+q_1}(t)\ b_{-q}(t) b^{+}_{q_1}(t);\ a^{+}_{\lambda}(t') \rangle\rangle$$

$$+ \sum_{\lambda_1} \phi_{\lambda_1(-q)} \langle\langle a_{\lambda+q}(t)\ a^{+}_{\lambda_1+q}(t)\ a_{\lambda_1}(t);\ a^{+}_{\lambda}(t') \rangle\rangle$$

$$- \sum_{q_1} \phi_{\lambda+q+q_1,q_1} \langle\langle a_{\lambda+q+q_1}(t)\ b_{-q_1}(t) b_{-q_1}(t);\ a^{+}_{\lambda}(t') \rangle\rangle$$

...... (2.38)

and

$$i\hbar \frac{d}{dt} \langle\langle a^{+}_{\lambda-q}(t) a_{\lambda}(t);\ b^{+}_{q}(t') \rangle\rangle$$

$$= (A_{\lambda} - A_{\lambda-q}) \langle\langle a^{+}_{\lambda-q}(t)\ a_{\lambda}(t);\ b^{+}_{q}(t) \rangle\rangle$$

$$+ \sum_{q_1} \phi_{\lambda+q_1,q_1} \langle\langle a^{+}_{\lambda-q}(t) a_{\lambda+q_1}(t) (b^{+}_{q_1}(t) - b_{-q_1}(t)); b^{+}_{q}(t') \rangle\rangle$$

$$- \sum_{q_1} \phi_{\lambda-q_1,q_1} \langle\langle a^{+}_{\lambda-q-q_1}(t) a_{\lambda}(t) (b^{+}_{q_1}(t) - b_{-q_1}(t)); b^{+}_{q}(t') \rangle\rangle$$

..... (2.39)

Before taking the energy Fourier transforms of the equations of motion noted above, the higher order Green's functions may be decoupled by making the following approximations:

$$\langle\langle a_{\lambda+q+q_1}(t)\; b_q^+(t) b_{-q_1}(t);\; a_\lambda^+(t') \rangle\rangle$$

$$= \delta_{q,-q_1} \langle b_q^+ \, b_{-q_1} \rangle \langle\langle a_{\lambda+q+q_1}(t);\; a_\lambda^+(t') \rangle\rangle \qquad (2.40)$$

$$\langle\langle a_{\lambda+q}(t) a^+_{\lambda_1+q_1}(t)\; a_{\lambda_1}(t);\; a_\lambda^+(t') \rangle\rangle$$

$$= \delta_{\lambda+q,\; \lambda_1+q_1} (1 + \langle a^+_{\lambda_1+q_1} a_{\lambda+q} \rangle) \langle\langle a_{\lambda_1}(t);\; a_\lambda^+(t') \rangle\rangle$$

etc. (2.41)

Terms involving factors like $b_q^+ b_{q_1}^+$ and $b_q b_{q_1}$ may be neglected. Thus the fourier energy transforms of the various equations of motion will be as follows:

$$(E - A_\lambda) \langle\langle a_\lambda;\; a_\lambda^+ \rangle\rangle_E = \frac{1}{2\pi} + \sum_q \phi_{\lambda+q,q} \langle\langle a_{\lambda+q}(b_q^+ - b_{-q});\; a_\lambda^+ \rangle\rangle_E$$

..... (2.42)

$$(E - \hbar w_q) \langle\langle b_q;\; b_q^+ \rangle\rangle_E = \frac{1}{2\pi} + \sum_\lambda \phi_{\lambda q} \langle\langle a^+_{\lambda-q}\; a_\lambda;\; b_q^+ \rangle\rangle_E \qquad (2.43)$$

$$(E - A_{\lambda+q} + \hbar w_q) \langle\langle a_{\lambda+q} b_q^+;\; a_\lambda^+ \rangle\rangle_E$$

$$= \phi_{\lambda,(-q)} (n_{\lambda+q} - N_q) \langle\langle a_\lambda;\; a_\lambda^+ \rangle\rangle_E \qquad (2.44)$$

$$(E + A_{\lambda+q} - \hbar w_q) \langle\langle a_{\lambda+q}\, b_{-q};\, a^+_\lambda \rangle\rangle_E$$

$$= \phi_{\lambda(-q)} (1+N_q+n_{\lambda+q}) \langle\langle a_\lambda;\, a^+_\lambda \rangle\rangle_E \qquad (2.45)$$

and

$$(E-A_\lambda + A_{\lambda-q}) \langle\langle a^+_{\lambda-q}\, a_\lambda;\, b^+_q \rangle\rangle_E$$

$$= -\phi_{(\lambda-q),(-q)} (n_\lambda - n_{\lambda-q}) \langle\langle b_q;\, b^+_q \rangle\rangle_E \qquad (2.46)$$

where N_q and n_λ are the phonon and magnon occupation numbers respectively.

On solving the coupled systems of equations (2.42) to (2.46) single particle Green's functions reduce to

$$\langle\langle a_\lambda;\, a^+_\lambda \rangle\rangle = \frac{1}{2\pi}\Bigg[E-A_\lambda + \sum_q \frac{\phi^2(\lambda-q)q\,(N_q-n_{\lambda-q})}{E-A_{\lambda-q}+\hbar w_q}$$

$$+ \sum_q \frac{(\phi_{\lambda,q})^2(1+N_q+n_{\lambda-q})}{E+A_{\lambda-q}-\hbar w_q}\Bigg]^{-1} \qquad (2.47)$$

$$\langle\langle b_q;\, b^+_q \rangle\rangle = \frac{1}{2\pi}\left[E-\hbar w_q + \sum_\lambda \frac{\phi^2_{\lambda q}(n_{\lambda-q}-n_\lambda)}{E-A_\lambda+A_{\lambda-q}}\right]^{-1}$$

$$\ldots\ldots (2.48)$$

Equations (2.47) and (2.48) can be re-expressed as

$$\frac{1}{2\pi} \langle\langle a_\lambda ; a_\lambda^+ \rangle\rangle^{-1} = E - A_\lambda - \sum_m (E_\lambda) \qquad (2.49)$$

$$\frac{1}{2\pi} \langle\langle b_q ; b_q^+ \rangle\rangle^{-1} = E - \hbar w_q - \sum_p (w_q) \qquad (2.50)$$

where $\sum_m (E_\lambda)$ and $\sum_p (w_q)$ are self-energy parts, complex in general.

The principal parts give the renormalisation of the respective single particle energies whereas the imaginary parts are related to the life-times of the quasi-particles. The explicit forms of the self-energy parts are

$$\sum_m (E_\lambda) = \sum_q \frac{|\phi_{\lambda q}|^2 (- n_{\lambda - q} + N_q)}{E - A_{\lambda - q} + \hbar w_q} + \sum_q |\phi_{\lambda q}^2| \frac{(1 + N_q + n_{\lambda - q})}{E + A_{\lambda - q} - \hbar w_q} \qquad (2.51)$$

and

$$\sum_p (w_q) = \sum_\lambda \frac{|\phi_{\lambda q}|^2 (n_{\lambda - q} - n_\lambda)}{(E - A_\lambda + A_{\lambda - q})} \qquad (2.52)$$

In the long wavelength approximation the coupling coefficient can be approximated as

$$|\phi_{\lambda q}|^2 = \frac{128}{N} \cdot \frac{\hbar}{Mv_s q} [S\ {}^eJ(R)]^2 [\lambda^2 q^2 a^4 \cos^2\theta_{\lambda q}]$$

.....(2.53)

The magnon concept being valid only in the low temperature limit, the expressions for self-energy parts worked out in this limit are given below. Detailed integration has been shown in Appendices A and B. It will be noted that in the expression for self-energy parts, energy values E and occupation number are replaced by the unperturbed values for the magnon and phonon cases. Thus we get after integration over $\underline{\lambda}$ and $\underline{q}$

$$\mathrm{Re} \sum_{m\lambda}(E) = -\frac{32}{3\pi^2} \frac{[S\ {}^eJ(R)]^2}{Mv_s^2} a^4 \left(\frac{k_B T}{2JS}\right)^{3/2} \Gamma\left(\frac{3}{2}\right)\lambda^2$$

$$\simeq -CT^{3/2}\lambda^2 \qquad (2.54)$$

where $\Gamma(3/2)$ is the Gamma function.

Thus for a cubic system, the renormalised magnon energy becomes

$$\tilde{A}_\lambda = (2JSa^2 - CT^{3/2})\lambda^2$$

$$\equiv D_o(1 - bT^{3/2})\lambda^2 \qquad (2.55)$$

where

$$D_o = 2JSa^2 \quad \text{and} \quad b = C/2JSa^2 \qquad (2.56)$$

Likewise

$$\mathrm{Re} \sum_{p}(w_q) = \frac{16}{3\pi^2} \left(\frac{5}{2}\right) . \left[\frac{k_B T}{2JS}\right]^{3/2} \frac{[S\ {}^{e}J(R_h)]^2\ a^4 q^2}{M v_s^2}$$

$$= F\ T^{3/2}\ q^2 \qquad (2.57)$$

and the renormalised accoustic mode phonon dispersion relation becomes

$$\hbar w_q = \hbar v_s q + F\ T^{3/2}\ q^2 \qquad (2.58)$$

Thus there is modification only in the second order of the propagation vector. This does not seem to be as strong as in the case of magnons. Of course, the predominant renormalisation effects on the phonon modes will arise from phonon-magnon interactions in the high temperature region i.e. above 20°K.

Next we calculate the life-time of magnon and phonon modes. These are

$$\frac{1}{\tau_{m,p}(\underline{\lambda},\underline{q})} = -\ \frac{2}{\hbar}\ \mathrm{Im} \sum_{m,\ p}(\lambda, q) \qquad (2.59)$$

Thus

$$\frac{1}{\tau_m(\lambda)} = \frac{2\pi}{\hbar} \sum_{q} |\phi_{\lambda \underline{q}}|^2 \Big\{ (N_{\underline{q}} - n_{\underline{\lambda - q}})\ \delta\ (E - A_{\underline{\lambda - q}} + \hbar w_{\underline{q}})$$

$$+ (1 + N_{\underline{q}} + n_{\underline{\lambda - q}})\ \delta\ (E + A_{\underline{\lambda - q}} - \hbar w_{\underline{q}}) \Big\} \qquad (2.60)$$

which on integration over $\underset{\sim}{q}$ (appendix C) gives in the low temperature approximation

$$\frac{1}{\tau_{m(\underset{\sim}{\lambda})}} = \frac{64\hbar}{3\pi} \frac{[S\, {}^{e}J(R_h)^2\, k_B \cdot \Theta_D}{M\, \Theta_C^3\, k_B^3} [\, 1+2\, e^{-\Theta_D^2/\Theta_c T}]\, a^2 \lambda 2 \qquad \ldots\ldots\ (2.61)$$

where we have used the notation

$$\Theta_D = \hbar v_s / k_B a \quad \text{and} \quad \Theta_c = 2JS/k_B$$

In obtaining (2.60) from (2.59) and (2.51), the use has been made of the Dirac identity

$$\lim_{\epsilon \to 0} \frac{1}{x + i\epsilon} = \frac{P}{x} - i\pi\delta(x) \qquad (2.62)$$

Likewise the phonon life-time is given by integration over $\underset{\sim}{\lambda}$ (Appendix D) as

$$\frac{1}{\tau_{p(\underset{\sim}{q})}} = \frac{32}{\pi} \frac{\hbar [\, S\, {}^{e}J(R_h)\,]^2\, k_B T}{M(k_B \Theta_D)(k_B \Theta_C)^2} \left[\frac{\Theta_D}{\Theta_C} + qa \right]^2 \times$$

$$\times \left[1 - e^{-\Theta_c/T} \left(\frac{\Theta_D}{\Theta_C} + qa \right)^2 \right] \times$$

$$\times\, e^{\Theta_c/T \left\{ (qa + \frac{\Theta_D}{\Theta_C})^2 - (\frac{\Theta_D}{\Theta_C})^2 - 1 \right\}} \qquad (2.63)$$

CHAPTER - 3

ANTIFERROMAGNETS

In this chapter, we consider the case of a simple two sub-lattice antiferromagnet, one of 'up' spin (S_l) and the other of 'down' spin (S_m).

Before writing the Hamiltonian for this case, a few words about the structure will be relevant. The possible orientational degeneracy of a ferromagnetic state could be removed by an external magnetic field. This cannot be done for an antiferromagnetic system in that there will be exchange alternation between two equivalent canonical states in the absence of anisotropy field. Hence it is essential to include the effective anisotropy field in the Hamiltonian. Thus we express the Hamiltonian for an antiferromagnetic system as[10]

$$H_{af} = \sum_{l,m} 2J(R_{l,m})\, \underline{S}_l \cdot \underline{S}_m - g\mu_B H \sum_{j=l,m} S_j^z$$

$$- H_A g\mu_B \left(\sum_l S_l^z - \sum_m S_m^z \right)$$

$$+ H_L + 8 \sum_{l \neq m} \left(\sum_h {}^e J(\underline{R}_{lm}) \cdot \delta \underline{R}_h \right) (\underline{S}_l \cdot \underline{S}_m) \qquad \ldots\ldots (3.1)$$

Here l spans the sub-lattice one and m spans the sublattice two, each having N magnetic atoms; H_A is the anisotropy field. Index j runs over both sub-lattice points. Other symbols have the same significance as in chapters 1 and 2.

The magnitude of the spin is assumed to be the same on all magnetic sites i.e. $|S_l| = |S_m| = S$.

Let us now introduce the following spin-wave transformations

$$\left.\begin{aligned} S_l^+ &= \left(\frac{2S}{N}\right)^{1/2} \sum_{\underline{\lambda}} e^{-i\underline{\lambda}.\underline{R}_l} a_{\underline{\lambda}} \\ S_l^- &= \left(\frac{2S}{N}\right)^{1/2} \sum_{\underline{\lambda}} e^{-i\underline{\lambda}.\underline{R}_l} a_{\underline{\lambda}}^+ \\ S-S_l^z &= \frac{1}{N} \sum_{\underline{\lambda}\underline{\lambda}'} e^{-i(\underline{\lambda}-\underline{\lambda}')R_l} a_{\underline{\lambda}}^+ a'_{\underline{\lambda}} \end{aligned}\right\} \quad (3.2)$$

and

$$\left.\begin{aligned} S_m^+ &= \left(\frac{2S}{N}\right)^{1/2} \sum_{\underline{\lambda}} e^{i\underline{\lambda}.\underline{R}_m} d_{\underline{\lambda}}^+ \\ S_m^- &= \left(\frac{2S}{N}\right)^{1/2} \sum_{\underline{\lambda}} e^{-i\underline{\lambda}.\underline{R}_m} d_{\underline{\lambda}} \\ S+S_m^z &= \frac{1}{N} \sum_{\underline{\lambda},\underline{\lambda}'} e^{-i(\underline{\lambda}-\underline{\lambda}')} d_{\underline{\lambda}}^+ d'_{\underline{\lambda}} \end{aligned}\right\} \quad (3.3)$$

where $a_{\underline{\lambda}}$ and $d_{\underline{\lambda}}$ are the two types of spin wave operators.

However, we have to introduce a further canonical transformation to make the magnon part diagonal and to write the interaction terms in the same representation. These are:

$$\left.\begin{aligned} a_{\underline{\lambda}} &= \alpha_{\underline{\lambda}} \cosh \Theta_{\underline{\lambda}} + \beta^{+}_{\underline{\lambda}} \sinh \Theta_{\underline{\lambda}} \\ d_{\underline{\lambda}} &= \alpha^{+}_{\underline{\lambda}} \sinh \Theta_{\underline{\lambda}} + \beta_{\underline{\lambda}} \cosh \Theta_{\underline{\lambda}} \end{aligned}\right\} \quad (3.4)$$

along with their Hermitian conjugates, where $(\alpha^{+}_{\underline{\lambda}}, \alpha_{\underline{\lambda}})$ and $(\beta^{+}_{\lambda}, \beta_{\underline{\lambda}})$ are the new magnon (creation and annihilation) operators. The angle parameter is given by

$$\tanh 2\Theta_{\underline{\lambda}} = - \frac{w_e \gamma_{\underline{\lambda}}}{w_e + w_A} \quad (3.5)$$

where

$$w_e = 2ZSJ/\hbar \quad (3.6)$$

$$w_A = g\mu_B H_A / \hbar \quad (3.7)$$

Thus our Hamiltonian (3.1) can be re-expressed as

$$H_{AF} = H_m + H_p + H_{int} \quad (3.8)$$

where

$$H_m = \sum_{\underline{\lambda}} \hbar w^{+}_{\underline{\lambda}} \left(\alpha^{+}_{\underline{\lambda}} \alpha_{\underline{\lambda}} + \frac{1}{2}\right) + \sum_{\underline{\lambda}} \hbar w^{-}_{\underline{\lambda}} \left(\beta^{+}_{\underline{\lambda}} \beta_{\underline{\lambda}} + \frac{1}{2}\right) \quad (3.9)$$

$$H_p = \sum_{\underline{q}} \hbar w_{\underline{q}} \left(b^{+}_{\underline{q}} b_{\underline{q}} + \frac{1}{2} \right) \tag{3.10}$$

and

$$H_{int} = \sum_{\underline{\lambda}\,\underline{q}} \Big[A_{\underline{\lambda}\underline{q}} \left(\alpha_{\underline{\lambda}} \alpha^{+}_{\underline{\lambda}-\underline{q}} b^{+}_{\underline{q}} - \alpha^{+}_{\underline{\lambda}} \alpha_{\underline{\lambda}-\underline{q}} b_{\underline{q}} \right)$$

$$+ B_{\underline{\lambda}\underline{q}} \left(\alpha_{\underline{\lambda}} \beta_{\underline{\lambda}-\underline{q}} b^{+}_{\underline{q}} - \alpha^{+}_{\underline{\lambda}} \beta^{+}_{\underline{\lambda}-\underline{q}} b_{\underline{q}} \right)$$

$$+ A_{\underline{\lambda}\underline{q}} \left(\beta_{\underline{\lambda}} \beta^{+}_{\underline{\lambda}-\underline{q}} b_{\underline{q}} - \beta^{+}_{\underline{\lambda}} \beta_{\underline{\lambda}-\underline{q}} b^{+}_{\underline{q}} \right) \Big] \tag{3.11}$$

where

$$\left.\begin{aligned} w^{\pm}_{\underline{\lambda}} &= \left[(w_e + w_A)^2 - w_e^2 \gamma^2_{\underline{\lambda}} \right] \pm w_H \\ w_H &= g\mu_B H / \hbar \end{aligned}\right\} \tag{3.12}$$

and $A_{\underline{\lambda}\underline{q}}$ and $B_{\underline{\lambda}\underline{q}}$ are the coupling coefficients having the form

$$A_{\lambda\underline{q}} = 4S \left({}^{e}\underline{J}(\underline{R}_{\underline{h}}) \cdot \underline{g}_{\underline{q}} \right) \left[(\gamma_{\underline{\lambda}-\underline{q}} - \gamma_{\underline{\lambda}}) \sinh (\theta_{\underline{\lambda}-\underline{q}} - \theta_{\underline{\lambda}}) \right.$$

$$\left. + (1 - \gamma_{\underline{q}}) \cosh (\theta_{\underline{\lambda}-\underline{q}} - \theta_{\underline{\lambda}}) \right]$$

$$B_{\lambda\underline{q}} = 4S \left({}^{e}\underline{J}(\underline{R}_{\underline{h}}) \cdot \underline{g}_{\underline{q}} \right) \left[(\gamma_{\underline{\lambda}-\underline{q}} - \gamma_{\underline{\lambda}}) \cosh (\theta_{\underline{\lambda}-\underline{q}} - \theta_{\underline{\lambda}}) \right.$$

$$\left. + (1 - \gamma_{\underline{q}}) \sinh (\theta_{\underline{\lambda}-\underline{q}} - \theta_{\underline{\lambda}}) \right] \tag{3.13}$$

As in the ferromagnetic case, we again follow the method of double-time temperature Green's functions for the two magnon modes and for the phonons.

These Green's functions are defined by

$$\langle\langle \alpha_{\lambda}(t);\ \alpha^{+}_{\lambda}(t')\rangle\rangle\ ,\quad \langle\langle \beta_{\lambda}(t);\ \beta^{+}_{\lambda}(t')\rangle\rangle \quad \text{for magnons}$$

and

$$\langle\langle b_{q}(t);\ b^{+}_{q}(t')\rangle\rangle \qquad \text{for the phonons.}$$

Their equations of motion are given as

$$i\hbar \frac{d}{dt}\langle\langle \alpha_{\lambda}(t);\ \alpha^{+}(t')\rangle\rangle = \hbar\delta(t-t')\ \langle [\alpha_{\lambda}, \alpha^{+}_{\lambda}]\rangle + \langle\langle [\alpha_{\lambda}(t),\ H(t)];\ \alpha^{+}_{\lambda}(t)\rangle\rangle \quad (3.14)$$

$$i\hbar \frac{d}{dt}\langle\langle \beta_{\lambda}(t);\beta^{+}_{\lambda}(t')\rangle\rangle = \hbar\delta(t-t')\langle [\beta_{\lambda},\beta^{+}_{\lambda}]\rangle + \langle\langle [\beta_{\lambda}(t);\ H(t)];\ \beta^{+}_{\lambda}(t')\rangle\rangle \quad (3.15)$$

and

$$i\hbar \frac{d}{dt}\langle\langle b_{q}(t);\ b^{+}_{q}(t')\rangle\rangle = \hbar\delta(t-t')\langle [b_{q}, b^{+}_{q}]\rangle + \langle\langle [b_{q}(t),\ H(t)];\ b^{+}_{q}(t')\rangle\rangle \quad (3.16)$$

Noting that magnons and phonons are bosons, the various commutators are as under:

$$[\alpha_{\underline{\lambda}}, \alpha^{+}_{\underline{\lambda}}] = 1 \tag{3.17}$$

$$[\alpha_{\underline{\lambda}}, H_m] = \hbar w^{+}_{\underline{\lambda}} \alpha_{\underline{\lambda}} \tag{3.18}$$

$$[\alpha_{\underline{\lambda}}, H_p] = 0 \tag{3.19}$$

$$[\alpha_{\underline{\lambda}}, H_{int}] = \sum_{\underline{q}} A_{(\underline{\lambda}+\underline{q})\underline{q}} \alpha_{\underline{\lambda}+\underline{q}} b^{+}_{\underline{q}} - \sum_{\underline{q}} A_{\underline{\lambda}\underline{q}} \alpha_{\underline{\lambda}-\underline{q}} b_{\underline{q}}$$

$$- \sum_{\underline{q}} B_{\underline{\lambda}\underline{q}} \beta^{+}_{\underline{\lambda}-\underline{q}} b_{\underline{q}} \tag{3.20}$$

$$[\beta_{\underline{\lambda}}; \beta^{+}_{\underline{\lambda}}] = 1 \tag{3.21}$$

$$[\beta_{\underline{\lambda}}, H_m] = \hbar w^{-}_{\underline{\lambda}} \beta_{\underline{\lambda}} \tag{3.22}$$

$$[\beta_{\underline{\lambda}}, H_p] = 0 \tag{3.23}$$

$$[\beta_{\underline{\lambda}}, H_{int}] = -\sum_{\underline{q}} B_{(\underline{\lambda}+\underline{q})\underline{q}} \alpha^{\dagger}_{\underline{\lambda}+\underline{q}} b_{\underline{q}} + \sum_{\underline{q}} A_{(\underline{\lambda}+\underline{q})\underline{q}} \beta_{\underline{\lambda}+\underline{q}} b_{\underline{q}}$$

$$- \sum_{\underline{q}} A_{\underline{\lambda}\underline{q}} \beta_{\underline{\lambda}-\underline{q}} b^{+}_{\underline{q}} \tag{3.24}$$

and

$$[b_{\underline{q}}, b^{+}_{\underline{q}}] = 1 \tag{3.25}$$

$$[b_{\underline{q}}, H_m] = 0 \tag{3.26}$$

$$[\, b_q, H_p \,] = \hbar w_q \, b_q \tag{3.27}$$

$$[\, b_q, H_{int}] = \sum_{\lambda} A_{\lambda q} \alpha_{\lambda} \alpha^{+}_{\lambda-q} + \sum_{\lambda} B_{\lambda q} \alpha_{\lambda} \beta_{\lambda-q}$$

$$- \sum_{\lambda} A_{\lambda q} \beta^{+}_{\lambda} \beta_{\lambda-q} \tag{3.28}$$

The equations of motion become

$$i\hbar \frac{d}{dt} \langle\langle \alpha_{\lambda}(t)+; \alpha^{+}_{\lambda}(t') \rangle\rangle =$$

$$= \hbar\delta(t-t') + \sum_{q} A_{(\lambda+q)q} \langle\langle \alpha_{\lambda+q} b^{+}_{q}(t); \alpha^{+}_{\lambda}(t') \rangle\rangle$$

$$- \sum_{q} A_{\lambda q} \langle\langle \alpha_{\lambda-q} b_q(t); \alpha^{+}_{\lambda}(t') \rangle\rangle$$

$$- \sum_{q} B_{\lambda q} \langle\langle \beta^{+}_{\lambda-q} b_q(t); \beta^{+}_{\lambda}(t') \rangle\rangle + \hbar w^{+}_{\lambda} \langle\langle \alpha_{\lambda}(t)+; \alpha^{+}_{\lambda}(t') \rangle\rangle$$

$$\ldots\ldots \tag{3.29}$$

$$i\hbar \frac{d}{dt} \langle\langle \beta_{\lambda}(t); \beta^{+}_{\lambda}(t') \rangle\rangle =$$

$$= \hbar\delta(t-t') - \sum_{q} B_{(\lambda+q)q} \langle\langle \alpha^{+}_{\lambda+q} b_q(t); \beta^{+}_{\lambda}(t') \rangle\rangle$$

$$+ \sum_{q} A_{(\lambda+q)q} \langle\langle \beta_{\lambda+q} b_q(t); \beta^{+}_{\lambda}(t') \rangle\rangle$$

$$- \sum_{q} A_{\lambda q} \langle\langle \beta_{\lambda-q} b^{+}_{q}(t); \beta^{+}_{\lambda}(t') \rangle\rangle + \hbar w^{-}_{\lambda} \langle\langle \beta_{\lambda}(t); \beta^{+}_{\lambda}(t') \rangle\rangle$$

$$\ldots\ldots \tag{3.30}$$

and

$$i\hbar \frac{d}{dt} \langle\langle b_{\underline{q}}(t); b_{\underline{q}}^{+}(t') \rangle\rangle =$$

$$= \hbar\delta(t-t') + \hbar w_{\underline{q}} \langle\langle b_{\underline{q}}(t); b_{\underline{q}}^{+}(t') \rangle\rangle + \sum_{\underline{\lambda}} A_{\underline{\lambda}\underline{q}} \langle\langle \alpha_{\underline{\lambda}} \alpha^{+}_{\underline{\lambda}-\underline{q}}(t); b_{\underline{q}}^{+}(t') \rangle\rangle$$

$$+ \sum_{\underline{\lambda}} B_{\underline{\lambda}\underline{q}} \langle\langle \alpha_{\underline{\lambda}} \beta_{\underline{\lambda}-\underline{q}}(t) b_{\underline{q}}^{+}(t') \rangle\rangle - \sum_{\underline{\lambda}} A_{\underline{\lambda}\underline{q}} \langle\langle \beta^{+}_{\underline{\lambda}} \beta_{\underline{\lambda}-\underline{q}}(t) \; ; b_{\underline{q}}^{+}(t') \rangle\rangle$$

.... (3.31)

The fourier energy transforms of the equation (3.29), (3.30) and (3.31) give the following set of coupled equations:

$$(E-\hbar w^{+}_{\underline{\lambda}}) \langle\langle \alpha_{\underline{\lambda}} ; \alpha^{+}_{\underline{\lambda}} \rangle\rangle_E = \frac{1}{2\pi} + \sum_{q} A_{(\underline{\lambda}+\underline{q})q} \langle\langle \alpha_{\underline{\lambda}+\underline{q}} b^{+}_{\underline{q}} ; \alpha^{+}_{\underline{\lambda}} \rangle\rangle_E$$

$$- \sum_{\underline{q}} A_{\underline{\lambda}\underline{q}} \langle\langle \alpha_{\underline{\lambda}-\underline{q}} b_{q} ; \alpha^{+}_{\underline{\lambda}} \rangle\rangle_E - \sum_{\underline{q}} B_{\underline{\lambda}\underline{q}} \langle\langle \beta^{+}_{\underline{\lambda}-\underline{q}} b_{\underline{q}} ; \alpha^{+}_{\underline{\lambda}} \rangle\rangle_E$$

.... (3.32)

$$(E-\hbar w^{-}_{\underline{\lambda}}) \langle\langle \beta_{\underline{\lambda}} ; \beta^{+}_{\underline{\lambda}} \rangle\rangle_E = \frac{1}{2\pi} \pm \sum_{q} B_{(\underline{\lambda}+\underline{q})q} \langle\langle \alpha_{\underline{\lambda}+\underline{q}} {}^{+} b_{\underline{q}} ; \beta^{+}_{\underline{\lambda}} \rangle\rangle_E$$

$$+ \sum_{\underline{q}} A_{(\underline{\lambda}+\underline{q})q} \langle\langle \beta_{\underline{\lambda}+\underline{q}} b_{\underline{q}} ; \beta^{+}_{\underline{\lambda}} \rangle\rangle_E - \sum_{\underline{q}} A_{\underline{\lambda}\underline{q}} \langle\langle \beta_{\underline{\lambda}-\underline{q}} b^{+}_{\underline{q}} ; \beta^{+}_{\underline{\lambda}} \rangle\rangle_E$$

.... (3.33)

and

$$(E-\hbar w_{q})\langle\langle b_{q}; b_{q}^{+}\rangle\rangle_{E} = \frac{1}{2\pi} + \sum_{\lambda} A_{\lambda q}\langle\langle \alpha_{\lambda}\alpha^{+}_{\lambda-q}; b^{+}_{q}\rangle\rangle_{E}$$

$$+ \sum_{\lambda} B_{\lambda q}\langle\langle \alpha_{\lambda}\beta_{\lambda-q}; b^{+}_{q}\rangle\rangle_{E} - \sum_{\lambda} A_{\lambda q}\langle\langle \beta^{+}_{\lambda}\beta_{\lambda-q}; b^{+}_{q}\rangle\rangle_{E}$$

.... (3.34)

In order to solve these equations, we have to write the equations of motion for each higher order Green's function on the right hand side of the above equations and to find the Fourier energy transform in each case.

The various commutators required for this purpose are as under:

$$[\alpha_{\lambda+q}b^{+}_{q}, \alpha^{+}_{\lambda}] = b^{+}_{q} \qquad (3.35)$$

$$[\alpha_{\lambda+q}b^{+}_{q}, H_{m}] = \hbar w^{+}_{\lambda+q}\alpha_{\lambda+q}b^{+}_{q} \qquad (3.36)$$

$$[\alpha_{\lambda+q}b^{+}_{q}, H_{p}] = -\hbar w_{q}\,\alpha_{\lambda+q}b^{+}_{q} \qquad (3.37)$$

$$[\alpha_{\lambda+q}b^{+}_{q}, H_{int}] = \sum_{q_1} A_{(\lambda+q+q_1)q_1}\alpha_{\lambda+q+q_1}b^{+}_{q}b^{+}_{q_1}$$

$$- A_{(\lambda+q),q}\,\alpha_{\lambda} - \sum_{q_1} A_{(\lambda+q),q_1}\alpha_{\lambda+q-q_1}b^{+}_{q}b_{q_1}$$

$$+ \sum_{\lambda_1} A_{\lambda_1,q}\,\alpha_{\lambda_1-q}\,\alpha_{\lambda+q}\,\alpha^{+}_{\lambda_1} - B_{(\lambda+q),q}\,\beta^{+}_{\lambda}$$

$$- \sum_{\underline{q}_1} B_{(\underline{\lambda}+\underline{q}),\underline{q}_1} \beta^{+}_{\underline{\lambda}+\underline{q}-\underline{q}_1} b^{+}_{\underline{q}_1} b_{\underline{q}_1}$$

$$+ \sum_{\underline{\lambda}_1} B_{\underline{\lambda}_1 \underline{q}} \beta^{+}_{\underline{\lambda}_1-\underline{q}} \alpha_{\underline{\lambda}+\underline{q}} \alpha^{+}_{\underline{\lambda}_1} - \sum_{\underline{\lambda}_1} A_{\underline{\lambda}_1 \underline{q}} \alpha_{\underline{\lambda}+\underline{q}} \beta_{\underline{\lambda}_1} \beta^{+}_{\underline{\lambda}_1-\underline{q}}$$

...... (3.38)

$$[\alpha_{\underline{\lambda}-\underline{q}} b_{\underline{q}}, \alpha^{+}_{\underline{\lambda}}] = b_{\underline{q}} \tag{3.39}$$

$$[\alpha_{\underline{\lambda}-\underline{q}} b_{\underline{q}}, H_m] = \hbar w^{+}_{\underline{\lambda}-\underline{q}} \alpha_{\underline{\lambda}-\underline{q}} b_{\underline{q}} \tag{3.40}$$

$$[\alpha_{\underline{\lambda}-\underline{q}} b_{\underline{q}}, H_p] = \hbar w_{\underline{q}} \alpha_{\underline{\lambda}-\underline{q}} b_{\underline{q}} \tag{3.41}$$

$$[\alpha_{\underline{\lambda}-\underline{q}} b_{\underline{q}}, H_{int}] = -A_{\underline{\lambda}\underline{q}} \alpha_{\underline{\lambda}} + \sum_{\underline{q}_1} A_{(\underline{\lambda}-\underline{q}+\underline{q}_1),\underline{q}_1} \alpha_{\underline{\lambda}-\underline{q}+\underline{q}_1} b_{\underline{q}} b^{+}_{\underline{q}_1}$$

$$+ \sum_{\underline{\lambda}_1} A_{\underline{\lambda}_1 \underline{q}} \alpha_{\underline{\lambda}_1} \alpha_{\underline{\lambda}-\underline{q}} \alpha^{+}_{\underline{\lambda}_1-\underline{q}} - \sum_{\underline{q}_1} A_{(\underline{\lambda}-\underline{q}),\underline{q}_1} \alpha_{\underline{\lambda}-\underline{q}-\underline{q}_1} b_{\underline{q}} b_{\underline{q}_1}$$

$$+ \sum_{\underline{\lambda}_1} B_{\underline{\lambda}_1 \underline{q}} \alpha_{\underline{\lambda}-\underline{q}} \alpha_{\underline{\lambda}_1} \beta_{\underline{\lambda}_1-\underline{q}} - \sum_{\underline{q}_1} B_{(\underline{\lambda}-\underline{q})\underline{q}_1} \beta^{+}_{\underline{\lambda}-\underline{q}-\underline{q}_1} b_{\underline{q}} b_{\underline{q}_1}$$

..... (3.42)

$$[\beta^{+}_{\underline{\lambda}-\underline{q}} b_{\underline{q}}, \alpha^{+}_{\underline{\lambda}}] = 0 \tag{3.43}$$

$$[\beta^{+}_{\underline{\lambda}-\underline{q}} b_{\underline{q}}, H_m] = -\hbar w^{-}_{\underline{\lambda}-\underline{q}} \beta^{+}_{\underline{\lambda}-\underline{q}} b_{\underline{q}} \tag{3.44}$$

$$[\beta^{+}_{\underline{\lambda}-\underline{q}} b_{\underline{q}}, H_p] = \hbar w_{\underline{q}} \beta^{+}_{\underline{\lambda}-\underline{q}} b_{\underline{q}} \tag{3.45}$$

$$[\beta^+_{\underline{\lambda}-\underline{q}} b_{\underline{q}}, H_{int}] =$$

$$= \sum_{\underline{\lambda}_1} A_{\underline{\lambda}_1 q} \beta^+_{\underline{\lambda}-\underline{q}} \alpha_{\underline{\lambda}_1} \alpha^+_{\underline{\lambda}_1-q} + B_{\underline{\lambda}\underline{q}} \alpha_{\underline{\lambda}} - \sum_{\underline{q}_1} B_{(\underline{\lambda}-\underline{q}+\underline{q}_1) q_1} \alpha_{\underline{\lambda}-\underline{q}+\underline{q}_1} b_{\underline{q}} b^+_{\underline{q}_1}$$

$$+ \sum_{\underline{\lambda}_1} B_{\underline{\lambda}_1 \underline{q}} \alpha_{\underline{\lambda}_1} \beta_{\underline{\lambda}-\underline{q}} + \beta_{\underline{\lambda}_1-\underline{q}} - \sum_{\underline{q}_1} A_{(\underline{\lambda}-\underline{q}) q_1} \beta^+_{\underline{\lambda}-\underline{q}-\underline{q}_1} b_{\underline{q}} b_{\underline{q}_1} - A_{\underline{\lambda} q} \beta^+_{\underline{\lambda}}$$

$$+ \sum_{\underline{q}_1} A_{(\underline{\lambda}-\underline{q}+\underline{q}_1) q_1} \beta^+_{\underline{\lambda}-\underline{q}+\underline{q}_1} b_{\underline{q}} b^+_{\underline{q}_1} - \sum_{\underline{\lambda}_1} A_{\underline{\lambda}_1 q} \beta^+_{\underline{\lambda}_1} \beta^+_{\underline{\lambda}-\underline{q}} \beta_{\underline{\lambda}_1-\underline{q}}$$

$$\ldots\ldots\ldots \quad (3.46)$$

$$[\alpha^+_{\underline{\lambda}+\underline{q}} b_{\underline{q}}, \beta^+_{\underline{\lambda}}] = 0 \quad (3.47)$$

$$[\alpha^+_{\underline{\lambda}+\underline{q}} b_{\underline{q}}, H_m] = -\hbar \varpi^+_{\underline{\lambda}+\underline{q}} \alpha^+_{\underline{\lambda}+\underline{q}} b_{\underline{q}} \quad (3.48)$$

$$[\alpha^+_{\underline{\lambda}+\underline{q}} b_{\underline{q}}, H_p] = \hbar w_{\underline{q}} \alpha^+_{\underline{\lambda}+\underline{q}} b_{\underline{q}} \quad (3.49)$$

$$[\alpha^+_{\underline{\lambda}+\underline{q}} b_{\underline{q}}, H_{int}] =$$

$$= A_{\underline{\lambda}+\underline{q}, q} \alpha^+_{\underline{\lambda}} + \sum_{\underline{\lambda}_1} A_{\underline{\lambda}_1 q_1} \alpha^+_{\underline{\lambda}_1-q} \alpha^+_{\underline{\lambda}+\underline{q}} \alpha_{\underline{\lambda}_1}$$

$$- \sum_{\underline{q}_1} A_{(\underline{\lambda}+\underline{q}) q_1} \alpha^+_{\underline{\lambda}+\underline{q}-\underline{q}_1} b_{\underline{q}} b^+_{\underline{q}_1}$$

$$+ \sum_{\underline{q}_1} A_{(\underline{\lambda}+\underline{q}+\underline{q}_1), q_1} \alpha^+_{\underline{\lambda}+\underline{q}+\underline{q}_1} b_{\underline{q}} b_{\underline{q}_1} + B_{(\underline{\lambda}+\underline{q}), q} \beta_{\underline{\lambda}}$$

$$+ \sum_{\underline{\lambda}_1} B_{\underline{\lambda}_1 q} \beta_{\underline{\lambda}_1-\underline{q}} \alpha^+_{\underline{\lambda}+\underline{q}} \alpha_{\underline{\lambda}_1} - \sum_{\underline{q}_1} B_{(\underline{\lambda}+\underline{q}), q_1} \beta_{\underline{\lambda}+\underline{q}-\underline{q}_1} b_{\underline{q}} b^+_{\underline{q}_1}$$

$$- \sum_{\underline{\lambda}_1} A_{\underline{\lambda}_1 q} \alpha^+_{\underline{\lambda}+\underline{q}} \beta^+_{\underline{\lambda}_1} \beta_{\underline{\lambda}_1-\underline{q}} \quad (3.50)$$

$$[\beta_{\underline{\lambda}+\underline{q}} b_{\underline{q}}, \beta^+_{\underline{\lambda}}] = b_{\underline{q}}\, \delta_{\underline{\lambda},\, \underline{\lambda}+\underline{q}} \tag{3.51}$$

$$[\beta_{\underline{\lambda}+\underline{q}} b_{\underline{q}}, H_m] = \hbar \bar{w}_{\underline{\lambda}+\underline{q}}\, \beta_{\underline{\lambda}+\underline{q}} b_{\underline{q}} \tag{3.52}$$

$$[\beta_{\underline{\lambda}+\underline{q}} b_{\underline{q}}, H_p] = \hbar w_{\underline{q}}\, \beta_{\underline{\lambda}+\underline{q}} b_{\underline{q}} \tag{3.53}$$

$$[\beta_{\underline{\lambda}+\underline{q}} b_{\underline{q}}, H_{int}] =$$

$$= \sum_{\underline{\lambda}_1} A_{\underline{\lambda}_1 q}\, \beta_{\underline{\lambda}+\underline{q}}\, \alpha_{\underline{\lambda}_1} \alpha^+_{\underline{\lambda}_1-\underline{q}} + \sum_{\underline{\lambda}_1} B_{\underline{\lambda}_1 q}\, \beta_{\underline{\lambda}+\underline{q}}\, \alpha_{\underline{\lambda}_1} \beta_{\underline{\lambda}_1-\underline{q}}$$

$$- \sum_{\underline{q}_1} B_{(\underline{\lambda}+\underline{q}+\underline{q}_1)q_1}\, \alpha^+_{\underline{\lambda}+\underline{q}+\underline{q}_1} b_{\underline{q}} b_{\underline{q}_1} + \sum_{\underline{q}_1} A_{(\underline{\lambda}+\underline{q}+\underline{q}_1)q_1}\, \beta_{\underline{\lambda}+\underline{q}+\underline{q}_1} b_{\underline{q}} b_{\underline{q}_1}$$

$$+ A_{(\underline{\lambda}+\underline{q}),q}\, \beta_{\underline{\lambda}} - \sum_{\underline{q}_1} A_{(\underline{\lambda}+\underline{q}),q_1}\, \beta_{\underline{\lambda}+\underline{q}-\underline{q}_1} b_{\underline{q}} b^+_{\underline{q}_1}$$

$$- \sum_{\underline{\lambda}_1} A_{\underline{\lambda}_1 q}\, \beta_{\underline{\lambda}+\underline{q}}\, \beta^+_{\underline{\lambda}_1}\, \beta_{\underline{\lambda}_1-\underline{q}} \tag{3.54}$$

$$[\beta_{\underline{\lambda}-\underline{q}} b^+_{\underline{q}}, \beta^+_{\underline{\lambda}}] = b^+_{\underline{q}}\, \delta_{\underline{\lambda}-\underline{q},\, \underline{\lambda}} \tag{3.55}$$

$$[\beta_{\underline{\lambda}-\underline{q}} b^+_{\underline{q}}, H_m] = \hbar \bar{w}_{\underline{\lambda}-\underline{q}}\, \beta_{\underline{\lambda}-\underline{q}} b^+_{\underline{q}} \tag{3.56}$$

$$[\beta_{\underline{\lambda}-\underline{q}} b^+_{\underline{q}}, H_p] = -\hbar w_{\underline{q}}\, \beta_{\underline{\lambda}-\underline{q}} b^+_{\underline{q}} \tag{3.57}$$

$$[\beta_{\underline{\lambda}-\underline{q}} b^{\pm}_{\underline{q}}, H_{int}] = \sum_{\underline{\lambda}_1} A_{\lambda_1 q}\, \beta_{\lambda-\underline{q}}\, \alpha^+_{\underline{\lambda}_1} \alpha_{\lambda-\underline{q}} - B_{\lambda q}\, \alpha^+_{\underline{\lambda}}$$

$$- \sum_{\underline{q}_1} B_{(\underline{\lambda}-\underline{q}+\underline{q}_1)q_1}\, \alpha^\dagger_{\lambda-\underline{q}+\underline{q}_1} b^+_{\underline{q}} b_{\underline{q}_1} + \sum_{\underline{\lambda}_1} B_{\lambda_1 q}\, \alpha_{\underline{\lambda}_1} \beta_{\underline{\lambda}-\underline{q}} \beta^+_{\underline{\lambda}_1-\underline{q}}$$

$$+ A_{\underline{\lambda}q}\beta_{\underline{\lambda}} + \sum_{q_1} A_{(\underline{\lambda}-\underline{q}+\underline{q}_1)q_1}\beta_{\underline{\lambda}-\underline{q}+\underline{q}_1} b^+_{\underline{q}} b_{\underline{q}_1}$$

$$- \sum_{\underline{\lambda}_1} A_{\underline{\lambda}_1 q}\beta_{\underline{\lambda}_1}\beta_{\underline{\lambda}-\underline{q}}\beta^+_{\underline{\lambda}_1-\underline{q}} - \sum_{q_1} A_{(\underline{\lambda}-\underline{q})q_1}\beta_{\underline{\lambda}-\underline{q}-\underline{q}_1} b^+_{\underline{q}_1} b^+_{\underline{q}} \quad \ldots\ldots\ (3.58)$$

$$[\alpha_{\underline{\lambda}}\alpha^+_{\underline{\lambda}-\underline{q}}, b_{\underline{q}}] = 0 \quad (3.59)$$

$$[\alpha_{\underline{\lambda}}\alpha^+_{\underline{\lambda}-\underline{q}}, H_m] = (\hbar w^+_{\underline{\lambda}} - hw^+_{\underline{\lambda}-\underline{q}})\,\alpha_{\underline{\lambda}}\alpha^+_{\underline{\lambda}-\underline{q}} \quad (3.60)$$

$$[\alpha_{\underline{\lambda}}\alpha^+_{\underline{\lambda}-\underline{q}}, H_p] = 0 \quad (3.61)$$

$$[\alpha_{\underline{\lambda}}\alpha^+_{\underline{\lambda}-\underline{q}}, H_{int}] = \sum_{\underline{q}_1} A_{(\underline{\lambda}+\underline{q}_1),q_1}\alpha_{\underline{\lambda}+\underline{q}_1}\alpha^+_{\underline{\lambda}-\underline{q}} b^+_{q_1}$$

$$- \sum_{q_1} A_{(\underline{\lambda}-\underline{q}),q_1}\alpha_{\underline{\lambda}}\alpha^+_{\underline{\lambda}-\underline{q}-\underline{q}_1} b^+_{q_1} - A_{\lambda\underline{q}} b_{\underline{q}}$$

$$- \sum_{q_1} A_{\lambda q_1}\alpha^+_{\underline{\lambda}-\underline{q}}\alpha_{\underline{\lambda}-\underline{q}_1} b_{q_1}$$

$$+ \sum_{q_1} A_{(\underline{\lambda}-\underline{q}+\underline{q}_1),q_1}\alpha_{\underline{\lambda}}\alpha^+_{\underline{\lambda}-\underline{q}+\underline{q}_1} b_{q_1}$$

$$- \sum_{q_1} B_{(\underline{\lambda}-\underline{q}),q_1}\alpha_{\underline{\lambda}}\beta_{\underline{\lambda}-\underline{q}-\underline{q}_1} b^+_{\underline{q}_1}$$

$$- \sum_{q_1} B_{\underline{\lambda}\underline{q}_1}\alpha^+_{\underline{\lambda}-\underline{q}}\beta^+_{\underline{\lambda}-\underline{q}_1} b_{\underline{q}_1} \quad (3.62)$$

$$[\alpha_{\underline{\lambda}}\beta_{\underline{\lambda}-\underline{q}}, b_{\underline{q}}] = 0 \quad (3.63)$$

$$[\alpha_{\underline{\lambda}}\beta_{\underline{\lambda}-\underline{q}},\ H_m\ ] = \hbar w^{+}_{\underline{\lambda}}\,\alpha_{\underline{\lambda}}\beta_{\underline{\lambda}-\underline{q}} + \hbar w^{-}_{\underline{\lambda}-\underline{q}}\alpha_{\underline{\lambda}}\beta_{\underline{\lambda}-\underline{q}} \tag{3.64}$$

$$[\alpha_{\underline{\lambda}}\beta_{\underline{\lambda}-\underline{q}}\ \ H_p\ ] = 0 \tag{3.65}$$

$$\begin{aligned}
[\alpha_{\underline{\lambda}}\beta_{\underline{\lambda}-\underline{q}},\ H_{int}\ ] = & \sum_{q_1} A_{(\underline{\lambda}+\underline{q})q_1}\,\alpha_{\underline{\lambda}+\underline{q}_1}\beta_{\underline{\lambda}-\underline{q}}\ b^{+}_{\underline{q}_1} \\
& - \sum_{q_1} A_{\underline{\lambda}q_1}\beta_{\underline{\lambda}-\underline{q}}\alpha_{\underline{\lambda}-\underline{q}_1}b_{q_1} + B_{\lambda\underline{q}}b_{\underline{q}} \\
& - \sum_{\underline{q}_1} B_{\underline{\lambda}\underline{q}_1}\ \beta_{\underline{\lambda}-\underline{q}}\ \beta^{+}_{\underline{\lambda}-q_1}\ b_{\underline{q}_1} \\
& - \sum_{\underline{q}_1} B_{(\underline{\lambda}-\underline{q}+\underline{q}_1),\underline{q}_1}\ \alpha_{\underline{\lambda}}\ \alpha^{+}_{\underline{\lambda}-\underline{q}+\underline{q}_1}\ b_{\underline{q}_1} \\
& + \sum_{\underline{q}_1} A_{(\underline{\lambda}-\underline{q}+\underline{q}_1),\underline{q}_1}\ \alpha_{\underline{\lambda}}\beta_{\underline{\lambda}-\underline{q}+\underline{q}_1}\ b_{\underline{q}_1} \\
& - \sum_{\underline{q}_1} A_{(\underline{\lambda}-\underline{q}),\underline{q}_1}\ \alpha_{\underline{\lambda}}\beta_{\underline{\lambda}-\underline{q}-\underline{q}_1}\ b^{+}_{\underline{q}_1}
\end{aligned} \tag{3.66}$$

$$\left[\beta^{+}_{\underline{\lambda}}\beta_{\underline{\lambda}-\underline{q}},\ b_{\underline{q}}\right] = 0 \tag{3.67}$$

$$[\beta^{+}_{\lambda}\beta_{\underline{\lambda}-\underline{q}},\ H_m\ ] = (\hbar w^{-}_{\underline{\lambda}-\underline{q}} - \hbar w^{-}_{\underline{\lambda}})\ \beta^{+}_{\underline{\lambda}}\ \beta_{\underline{\lambda}-\underline{q}} \tag{3.68}$$

$$[\beta^{+}_{\lambda}\beta_{\underline{\lambda}-\underline{q}},\ H_p\ ] = 0 \tag{3.69}$$

$$\begin{aligned}
[\beta^{+}_{\underline{\lambda}}\beta_{\underline{\lambda}-\underline{q}},\ H_{int}] = & -\sum_{\underline{q}_1} B_{(\underline{\lambda}+\underline{q}_1),\underline{q}_1}\ \alpha_{\underline{\lambda}+\underline{q}_1}\ \beta_{\underline{\lambda}-\underline{q}}\ b^{+}_{\underline{q}_1} \\
& - \sum_{\underline{q}_1} B_{(\underline{\lambda}+\underline{q}_1-\underline{q}),\underline{q}_1}\ \beta^{+}_{\underline{\lambda}}\ \alpha^{+}_{\underline{\lambda}+\underline{q}_1-\underline{q}}\ b_{\underline{q}_1} + A_{\lambda\underline{q}}\ b_{\underline{q}}
\end{aligned}$$

$$- \sum_{q_1} A_{\lambda q_1} \beta_{\lambda-q} \beta^{+}_{\lambda-q_1} b_{q_1} + \sum_{q_1} A_{(\lambda+q+q_1),q_1} \beta^{+}_{\lambda} \beta_{\lambda-q+q_1} b_{q_1}$$

$$+ \sum_{q_1} A_{(\lambda+q),q_1} \beta_{\lambda+q-q_1} \beta_{\lambda-q} - \sum_{q_1} A_{(\lambda-q)q_1} \beta^{+}_{\lambda} \beta_{\lambda-q-q_1} b^{+}_{q_1}$$

$$\ldots\ldots \quad (3.70)$$

Noting the above commutators and taking the fourier energy transforms of the equations of motion of the relevant higher order Green's functions, we obtain the following sets of coupled equations for them:

$$(E+\hbar w_q-\hbar w^{+}_{\lambda+q}) \langle\langle \alpha_{\lambda+q} b^{+}_{q}; \alpha^{+}_{\lambda} \rangle\rangle = \langle b^{+}_{q} \rangle / 2\pi +$$

$$+ \sum_{q_1} A_{(\lambda+q+q_1)q_1} \langle\langle \alpha_{\lambda+q+q_1} b^{+}_{q} b^{+}_{q_1} ; \alpha^{+}_{\lambda} \rangle\rangle - A_{\lambda+q,q} \langle\langle \alpha_{\lambda}; \alpha^{+}_{\lambda} \rangle\rangle$$

$$- \sum_{q_1} A_{(\lambda+q),q_1} \langle\langle \alpha_{\lambda+q-q_1} b^{+}_{q} b_{q_1}; \alpha^{+}_{\lambda} \rangle\rangle$$

$$+ \sum_{\lambda_1} A_{\lambda_1 q} \langle\langle \alpha_{\lambda_1-q} \alpha_{\lambda+q} \alpha^{+}_{\lambda_1}; \alpha^{+}_{\lambda} \rangle\rangle - B_{\lambda+q,q} \langle\langle \beta^{+}_{\lambda}; \alpha^{+}_{\lambda} \rangle\rangle$$

$$- \sum_{q_1} B_{(\lambda+q),q_1} \langle\langle \beta_{\lambda+q-q_1} b^{+}_{q_1} b_{q}; \alpha^{+}_{\lambda} \rangle\rangle$$

$$+ \sum_{\lambda_1} B_{\lambda_1 q} \langle\langle \beta^{+}_{\lambda_1-q} \alpha_{\lambda+q} \alpha^{+}_{\lambda_1}; \alpha^{+}_{\lambda} \rangle\rangle$$

$$+ \sum_{\lambda_1} A_{\lambda_1 q} \langle\langle \alpha_{\lambda+q} \beta_{\lambda_1} \beta^{+}_{\lambda_1-q}; \alpha^{+}_{\lambda} \rangle\rangle \quad (3.71)$$

$$(E-\hbar w_{\underline{q}}-\hbar w^{+}_{\underline{\lambda}-\underline{q}})\langle\langle \alpha_{\underline{\lambda}-\underline{q}} b_{\underline{q}}; \alpha^{+}_{\underline{\lambda}}\rangle\rangle = \frac{\langle b_{\underline{q}}\rangle}{2\pi} + A_{\underline{\lambda}\underline{q}}\langle\langle \alpha_{\underline{\lambda}} ; \alpha^{+}_{\underline{\lambda}}\rangle\rangle$$

$$- \sum_{\underline{q}_1} A_{(\underline{\lambda}-\underline{q}+\underline{q}_1)\underline{q}}\langle\langle \alpha_{\underline{\lambda}-\underline{q}+\underline{q}_1} b_{\underline{q}} b^{+}_{\underline{q}_1} ; \alpha^{+}_{\underline{\lambda}}\rangle\rangle$$

$$+ \sum_{\underline{\lambda}_1} A_{\underline{\lambda}_1\underline{q}}\langle\langle \alpha_{\underline{\lambda}_1} \alpha_{\underline{\lambda}-\underline{q}} \alpha^{+}_{\underline{\lambda}_1-\underline{q}_1} ; \alpha^{+}_{\underline{\lambda}}\rangle\rangle$$

$$- \sum_{\underline{q}_1} A_{(\underline{\lambda}-\underline{q}),\underline{q}_1}\langle\langle \alpha_{\underline{\lambda}-\underline{q}-\underline{q}_1} b_{\underline{q}} b_{\underline{q}_1} ; \alpha^{+}_{\underline{\lambda}}\rangle\rangle$$

$$+ \sum_{\underline{\lambda}_1} B_{\underline{\lambda}_1\underline{q}}\langle\langle \alpha_{\underline{\lambda}-\underline{q}} \alpha_{\underline{\lambda}_1} \beta_{\underline{\lambda}_1-\underline{q}} ; \alpha^{+}_{\underline{\lambda}}\rangle\rangle$$

$$- \sum_{\underline{q}_1} B_{(\underline{\lambda}-\underline{q}),\underline{q}_1}\langle\langle \beta^{+}_{\underline{\lambda}-\underline{q}-\underline{q}_1} b_{\underline{q}} b_{\underline{q}_1} ; \alpha^{+}_{\underline{\lambda}}\rangle\rangle \qquad (3.72)$$

$$(E-\hbar w_{\underline{q}}+\hbar w^{-}_{\underline{\lambda}-\underline{q}})\langle\langle \beta^{+}_{\underline{\lambda}-\underline{q}} b_{\underline{q}} ; \alpha^{+}_{\underline{\lambda}}\rangle\rangle = \sum_{\underline{\lambda}_1} A_{\underline{\lambda}_1\underline{q}}\langle\langle \beta^{+}_{\underline{\lambda}-\underline{q}} \alpha_{\underline{\lambda}_1} \alpha^{+}_{\underline{\lambda}_1-\underline{q}} ; \alpha^{+}_{\underline{\lambda}}\rangle\rangle$$

$$+ B_{\underline{\lambda}\underline{q}}\langle\langle \alpha_{\underline{\lambda}} ; \alpha^{+}_{\underline{\lambda}}\rangle\rangle - \sum_{\underline{q}_1} B_{(\underline{\lambda}-\underline{q}+\underline{q}_1)\underline{q}_1}\langle\langle \alpha_{\underline{\lambda}-\underline{q}+\underline{q}_1} b_{\underline{q}} b^{+}_{\underline{q}_1} ; \alpha^{+}_{\underline{\lambda}}\rangle\rangle$$

$$+ \sum_{\underline{\lambda}_1} B_{\underline{\lambda}_1\underline{q}}\langle\langle \alpha_{\underline{\lambda}_1} \beta^{+}_{\underline{\lambda}-\underline{q}} \beta_{\underline{\lambda}_1-\underline{q}} ; \alpha^{+}_{\underline{\lambda}}\rangle\rangle - \sum_{\underline{q}_1} A_{(\underline{\lambda}-\underline{q}),\underline{q}_1}\langle\langle \beta^{+}_{\underline{\lambda}-\underline{q}-\underline{q}_1} b_{\underline{q}} b_{\underline{q}_1} ; \alpha^{+}_{\underline{\lambda}}\rangle\rangle$$

$$- A_{\underline{\lambda}\underline{q}}\langle\langle \beta^{+}_{\underline{\lambda}} ; \alpha^{+}_{\underline{\lambda}}\rangle\rangle + \sum_{\underline{q}_1} A_{(\underline{\lambda}-\underline{q}+\underline{q}_1)\underline{q}_1}\langle\langle \beta_{\underline{\lambda}-\underline{q}+\underline{q}_1} b_{\underline{q}} b^{+}_{\underline{q}_1} ; \alpha^{+}_{\underline{\lambda}}\rangle\rangle$$

$$- \sum_{\underline{\lambda}_1} A_{\underline{\lambda}_1\underline{q}}\langle\langle \beta^{+}_{\underline{\lambda}_1} \beta^{+}_{\underline{\lambda}-\underline{q}} \beta_{\underline{\lambda}_1-\underline{q}} ; \alpha^{+}_{\underline{\lambda}}\rangle\rangle \qquad (3.73)$$

$$(E-\hbar w_{q}+\hbar w^{+}_{\lambda+q})\langle\langle\alpha^{+}_{\lambda+q}b_{q};\ \beta^{+}_{\lambda}\rangle\rangle = A_{(\lambda+q),q}\langle\langle\alpha^{+}_{\lambda};\ \beta^{+}_{\lambda}\rangle\rangle$$

$$+\sum_{\lambda_1}A_{\lambda_1 q}\langle\langle\alpha^{+}_{\lambda_1-q}\alpha^{+}_{\lambda+q}\alpha_{\lambda_1};\beta^{+}_{\lambda}\rangle\rangle-\sum_{q_1}A_{(\lambda+q)q_1}\langle\langle\alpha^{+}_{\lambda+q-q_1}b_{q}b^{+}_{q_1};\beta^{+}_{\lambda}\rangle\rangle$$

$$+\sum_{q_1}A_{(\lambda+q+q_1),q_1}\langle\langle\alpha_{\lambda+q+q_1}b_{q}b_{q_1};\ \beta^{+}_{\lambda}\rangle\rangle+B_{\lambda+q,q}\langle\langle\beta_{\lambda};\beta^{+}_{\lambda}\rangle\rangle$$

$$+\sum_{\lambda_1}B_{\lambda_1 q}\langle\langle\beta_{\lambda_1-q}\alpha^{+}_{\lambda+q}\alpha_{\lambda_1};\beta^{+}_{\lambda}\rangle\rangle-\sum_{q_1}B_{(\lambda+q)q_1}\langle\langle\beta_{\lambda+q-q_1}b_{q}b^{+}_{q_1};\beta^{+}_{\lambda}\rangle\rangle$$

$$-\sum_{\lambda_1}A_{\lambda_1 q}\langle\langle\alpha^{+}_{\lambda+q}\beta^{+}_{\lambda_1}\beta_{\lambda_1-q};\ \beta^{+}_{\lambda}\rangle\rangle \qquad (3.74)$$

$$(E-\hbar w_{q}-\hbar w^{-}_{\lambda+q})\langle\langle\beta_{\lambda+q}b_{q};\beta^{+}_{\lambda}\rangle\rangle=\frac{\langle b_{q}\rangle}{2\pi}+\sum_{\lambda_1}A_{\lambda_1 q}\langle\langle\beta_{\lambda+q}\alpha_{\lambda_1}\alpha^{+}_{\lambda_1-q};\beta^{+}_{\lambda}\rangle\rangle$$

$$+\sum_{\lambda_1}B_{\lambda_1 q}\langle\langle\beta_{\lambda+q}\alpha_{\lambda_1}\beta_{\lambda_1-q};\beta^{+}_{\lambda}\rangle\rangle-\sum_{q_1}B_{(\lambda+q+q_1)q_1}\langle\langle\alpha^{+}_{\lambda+q+q_1}b_{q}b_{q_1};\beta^{+}_{\lambda}\rangle\rangle$$

$$+\sum_{q_1}A_{(\lambda+q+q_1),q_1}\langle\langle\beta_{\lambda+q+q_1}b_{q}b_{q_1};\ \beta^{+}_{\lambda}\rangle\rangle+A_{\lambda+q,q}\langle\langle\beta_{\lambda};\beta^{+}_{\lambda}\rangle\rangle$$

$$-\sum_{q_1}A_{(\lambda+q)q_1}\langle\langle\beta_{\lambda+q-q_1}b_{q}b^{+}_{q_1};\beta^{+}_{\lambda}\rangle\rangle-\sum_{\lambda_1}A_{\lambda_1 q}\langle\langle\beta_{\lambda+q}\beta^{+}_{\lambda_1}\beta_{\lambda_1-q};\beta^{+}_{\lambda}\rangle\rangle$$

$$\dots\ \dots \qquad (3.75)$$

$$(E+\hbar w_{q}-\hbar w^{-}_{\lambda-q})\langle\langle\beta_{\lambda-q}b^{+}_{q};\ \beta^{+}_{\lambda}\rangle\rangle = \frac{\langle b_{q}\rangle}{2\pi}$$

$$+\sum_{\lambda_1}A_{\lambda_1 q}\langle\langle\beta_{\lambda-q}\alpha^{+}_{\lambda_1}\alpha_{\lambda-q};\ \beta^{+}_{\lambda}\rangle\rangle-B_{\lambda q}\langle\langle\alpha^{+}_{\lambda};\ \beta^{+}_{\lambda}\rangle\rangle$$

$$- \sum_{q_1} B_{(\lambda-q+q_1)q_1} \langle\langle \alpha^{+}_{\lambda-q+q_1} b^{+}_{q} b_{q_1} ; \beta^{+}_{\lambda} \rangle\rangle + \sum_{\lambda_1} B_{\lambda_1 q} \langle\langle \alpha_{\lambda_1} \beta_{\lambda-q} \beta^{+}_{\lambda_1-q} ; \beta^{+}_{\lambda} \rangle\rangle$$

$$+ A_{\lambda q} \langle\langle \beta_{\lambda} ; \beta^{+}_{\lambda} \rangle\rangle + \sum_{q_1} A_{(\lambda-q+q_1),q_1} \langle\langle \beta_{\lambda-q+q_1} b^{+}_{q} b_{q_1} ; \beta^{+}_{\lambda} \rangle\rangle$$

$$- \sum_{\lambda_1} A_{\lambda_1 q} \langle\langle \beta_{\lambda_1} \beta_{\lambda-q} \beta^{+}_{\lambda_1-q} ; \beta^{+}_{\lambda} \rangle\rangle - \sum_{q_1} A_{(\lambda-q),q_1} \langle\langle \beta_{\lambda-q-q_1} b^{+}_{q_1} b^{+}_{q_1} ; \beta^{+}_{\lambda} \rangle\rangle$$

.... (3.76)

$$(E-\hbar w^{+}_{\lambda}+\hbar w^{+}_{\lambda-q}) \langle\langle \alpha_{\lambda} \alpha^{+}_{\lambda-q} ; b^{+}_{q} \rangle\rangle = \sum_{q_1} A_{(\lambda+q_1),q_1} \langle\langle \alpha_{\lambda+q_1} \alpha^{+}_{\lambda-q} b^{+}_{q_1} ; b^{+}_{q} \rangle\rangle$$

$$- \sum_{q_1} A_{(\lambda-q),q_1} \langle\langle \alpha_{\lambda} \alpha^{+}_{\lambda-q-q_1} b^{+}_{q_1} ; b^{+}_{q} \rangle\rangle - A_{\lambda q} \langle\langle b_{q} ; b^{+}_{q} \rangle\rangle$$

$$- \sum_{q_1} A_{\lambda q_1} \langle\langle \alpha^{+}_{\lambda-q} \alpha_{\lambda-q_1} b_{q_1} ; b^{+}_{q} \rangle\rangle + \sum_{q_1} A_{(\lambda-q+q_1)q_1} \langle\langle \alpha_{\lambda} \alpha^{+}_{\lambda+q-q_1} b_{q_1} ; b^{+}_{q} \rangle\rangle$$

$$- \sum_{q_1} B_{(\lambda-q)q_1} \langle\langle \alpha_{\lambda} \beta_{\lambda-q-q_1} b^{+}_{q_1} ; b^{+}_{q} \rangle\rangle - \sum_{q_1} B_{\lambda q_1} \langle\langle \alpha^{+}_{\lambda-q} \beta^{+}_{\lambda-q_1} b_{q_1} ; b^{+}_{q} \rangle\rangle$$

.. ... (3.77)

$$(E-\hbar w^{+}_{\lambda}-\hbar w^{-}_{\lambda-q}) \langle\langle \alpha_{\lambda} \beta_{\lambda-q} ; b^{+}_{q} \rangle\rangle = \sum_{q_1} A_{(\lambda+q),q_1} \langle\langle \alpha_{\lambda+q_1} \beta_{\lambda-q} b^{+}_{q_1} ; b^{+}_{q} \rangle\rangle$$

$$- \sum_{q_1} A_{\lambda q_1} \langle\langle \beta_{\lambda-q} \alpha_{\lambda-q_1} b_{q_1} ; b^{+}_{q} \rangle\rangle + B_{\lambda q} \langle\langle b_{q} ; b^{+}_{q} \rangle\rangle -$$

$$- \sum_{q_1} B_{\lambda q_1} \langle\langle \beta_{\lambda-q} \beta^{+}_{\lambda-q_1} b^{+}_{q_1} ; b^{+}_{q} \rangle\rangle - \sum_{q_1} B_{(\lambda-q+q_1),q_1} \langle\langle \alpha_{\lambda} \alpha^{+}_{\lambda-q+q_1} b_{q_1} ; b^{+}_{q} \rangle\rangle$$

$$+ \sum_{q_1} A_{(\lambda-q+q_1)q_1} \langle\langle \alpha_{\lambda} \beta_{\lambda-q+q_1} b_{q_1} ; b^{+}_{q} \rangle\rangle -$$

$$- \sum_{q_1} A_{(\lambda-q),q_1} \langle\langle \alpha_{\lambda} \beta_{\lambda-q-q_1} b^{+}_{q_1} ; b^{+}_{q} \rangle\rangle \qquad (3.78)$$

$$[E-\hbar\omega^{-}_{\lambda-q}+\hbar\omega^{-}_{\lambda}]\langle\langle \beta^{+}_{\lambda}\beta_{\lambda-q};b^{+}_{q}\rangle\rangle=-\sum_{q_1}B_{(\lambda+q_1)q_1}\langle\langle \alpha_{\lambda+q_1}\beta_{\lambda-q}b^{+}_{q_1};b^{+}_{q}\rangle\rangle$$

$$-\sum_{q_1}B_{(\lambda+q_1-q),q_1}\langle\langle \beta^{+}_{\lambda}\alpha^{+}_{\lambda+q_1-q}b_{q_1};b^{+}_{q}\rangle\rangle+A_{\lambda q}\langle\langle b_{q};b^{+}_{q}\rangle\rangle$$

$$-\sum_{q_1}A_{\lambda q_1}\langle\langle \beta_{\lambda-q}\beta^{+}_{\lambda-q_1}b_{q_1};b^{+}_{q}\rangle\rangle$$

$$+\sum_{q_1}A_{(\lambda+q-q_1),q_1}\langle\langle \beta^{+}_{\lambda}\beta_{\lambda-q+q_1}b_{q_1};b^{+}_{q}\rangle\rangle$$

$$+\sum_{q_1}A_{(\lambda+q),q_1}\langle\langle \beta_{\lambda+q_1}\beta_{\lambda-q}b_{q_1};b^{+}_{q}\rangle\rangle$$

$$-\sum_{q_1}A_{(\lambda-q)q_1}\langle\langle \beta^{+}_{\lambda}\beta_{\lambda-q-q_1}b^{+}_{q_1};b^{+}_{q}\rangle\rangle \qquad (3.79)$$

As in the ferromagnetic case, the following type of decoupling can be used:

$$\langle\langle \alpha_{\lambda-q+q_1}b_{q}b^{+}_{q_1};\alpha^{+}_{\lambda}\rangle\rangle=\delta_{qq_1}\langle b_{q}b^{+}_{q_1}\rangle\langle\langle \alpha_{\lambda-q+q_1};\alpha^{+}_{\lambda}\rangle\rangle \qquad (3.80)$$

$$\langle\langle \alpha_{\lambda_1}\alpha_{\lambda-q}\alpha^{+}_{\lambda_1-q};\alpha^{+}_{\lambda}\rangle\rangle=\delta_{\lambda\lambda_1}\langle \alpha_{\lambda-q}\alpha^{+}_{\lambda_1-q}\rangle\langle\langle \alpha_{\lambda};\alpha^{+}_{\lambda}\rangle\rangle \qquad (3.81)$$

etc.

Furthermore, Green's functions of the type

$$\langle\langle \alpha_{\lambda-q-q_1} b_q b_{q_1} \; ; \; \alpha^+_\lambda \rangle\rangle \simeq 0$$

$$\langle\langle \alpha_{\lambda+q_1} \beta_{\lambda-q} b^+_{q_1} \; ; \; b^+_q \rangle\rangle \simeq 0 \qquad (3.82)$$

and $$\langle\langle \beta_{\lambda+q-q_1} b^+_{q_1} b_q \; ; \; \alpha^+_\lambda \rangle\rangle \simeq 0$$

have been neglected as there is no direct interaction between the magnon modes within the Hamiltonian described in equation (3.1) and (3.8) to (3.12).

Also the values of $\langle b^+_q \rangle$ and $\langle b_q \rangle$ have been assumed to be zero.

With these approximations and using the occupation number representation i.e.

$$\langle \alpha^+_\lambda \alpha_\lambda \rangle = n^\alpha_\lambda$$

$$\langle \beta^+_\lambda \beta_\lambda \rangle = n^\beta_\lambda \qquad (3.83)$$

and

$$\langle b^+_q b_q \rangle = N_q$$

We obtain the following equations for the higher order Green's functions:

$$(E-\hbar w^+_{\lambda-q}-\hbar w_q) \langle\langle \alpha_{\lambda-q} b_q ; \alpha^+_\lambda \rangle\rangle = A_{\lambda q}(1+n^\alpha_{\lambda-q} \pm N_q) \langle\langle \alpha_\lambda ; \alpha^+_\lambda \rangle\rangle$$

.... (3.84)

$$(E-\hbar w^{+}_{\lambda+q}+\hbar w_{q})\langle\langle \alpha_{\lambda+q} b^{+}_{q};\ \alpha^{+}_{\lambda}\rangle\rangle = -A_{(\lambda+q),q}(1+N_{q})\langle\langle \alpha_{\lambda};\alpha^{+}_{\lambda}\rangle\rangle \quad \ldots\ldots (3.85)$$

$$(E+\hbar w^{-}_{\lambda-q}-\hbar w_{q})\langle\langle \beta^{+}_{\lambda-q} b_{q};\alpha^{+}_{\lambda}\rangle\rangle = B_{\lambda q}(n^{\beta}_{\lambda-q}-N_{q})\langle\langle \alpha_{\lambda};\ \alpha^{+}_{\lambda}\rangle\rangle \quad \ldots\ldots (3.86)$$

Likewise:

$$(E+\hbar w^{+}_{\lambda+q}-\hbar w_{q})\langle\langle \alpha^{+}_{\lambda+q} b_{q};\beta^{+}_{\lambda}\rangle\rangle = -\ B_{(\lambda+q),q} N_{q}\langle\langle \beta_{\lambda};\beta^{+}_{\lambda}\rangle\rangle \quad (3.87)$$

$$(E-\hbar w^{-}_{\lambda+q}-\hbar w_{q})\langle\langle \beta_{\lambda+q} b_{q};\beta^{+}_{\lambda}\rangle\rangle = -\ A_{(\lambda+q),q} N_{q}\langle\langle \beta_{\lambda};\beta^{+}_{\lambda}\rangle\rangle \quad (3.88)$$

$$(E-\hbar w^{-}_{\lambda-q}+\hbar w_{q})\langle\langle \beta_{\lambda-q} b^{+}_{q};\beta^{+}_{\lambda}\rangle\rangle = A_{\lambda q}(N_{q}-n^{\beta}_{\lambda-q})\langle\langle \beta_{\lambda};\beta^{+}_{\lambda}\rangle\rangle \quad \ldots\ldots\ldots (3.89)$$

and

$$(E-\hbar w^{+}_{\lambda}+\hbar w^{+}_{\lambda-q})\langle\langle \alpha_{\lambda}\alpha^{+}_{\lambda-q};b^{+}_{q}\rangle\rangle = A_{\lambda q}(n^{\alpha}_{\lambda}-n^{\alpha}_{\lambda-q})\langle\langle b_{q};b^{+}_{q}\rangle\rangle \quad \ldots\ldots\ldots (3.90)$$

$$(E-\hbar w^{+}_{\lambda}-\hbar w^{-}_{\lambda-q})\langle\langle \alpha_{\lambda}\beta_{\lambda-q};b^{+}_{q}\rangle\rangle = -\ B_{\lambda q}(1+n^{\alpha}_{\lambda}+n^{\beta}_{\lambda-q})\langle\langle b_{q};\ b^{+}_{q}\rangle\rangle \quad \ldots\ \ldots (3.91)$$

$$(E-\hbar w^{-}_{\lambda-q}+\hbar w^{-}_{\lambda})\langle\langle \beta^{+}_{\lambda}\beta_{\lambda-q};b^{+}_{q}\rangle\rangle = -\ A_{\lambda q}\cdot n^{\beta}_{\lambda-q}\langle\langle b_{q};\ b^{+}_{q}\rangle\rangle \quad (3.92)$$

Using the above to solve the set of coupled equations (3.32) to (3.34) we have

$$\langle\langle \alpha_{\lambda};\alpha^{+}_{\lambda}\rangle\rangle = \frac{1}{2\pi}\Big[E-\hbar w^{+}_{\lambda} + \sum_{q} \frac{A^{2}_{(\lambda+q),q}}{E-\hbar w^{+}_{\lambda+q}+\hbar w_{q}}(1+N_{q})$$

$$+\sum_{q} \frac{A^{2}_{\lambda q}}{E-\hbar w^{+}_{\lambda-q}-\hbar w_{q}}(1+n^{\alpha}_{\lambda-q}-N_{q})$$

$$+\sum_{q} \frac{B^{2}_{\lambda q}}{E+\hbar w^{-}_{\lambda-q}-\hbar w_{q}}(n^{\beta}_{\lambda-q}-N_{q})\Big]^{-1}$$

Therefore

$$\frac{1}{2\pi}\langle\langle \alpha_{\lambda};\alpha^{+}_{\lambda}\rangle\rangle^{-1} \equiv (E-\hbar w^{+}_{\lambda} - \sum\nolimits_{m}^{(\alpha)} E(\lambda) \qquad (3.93)$$

$$\langle\langle \beta_{\lambda};\beta^{+}_{\lambda}\rangle\rangle = \frac{1}{2\pi}\Big[E-\hbar w^{-}_{\lambda} + \sum_{q} \frac{B^{2}_{(\lambda+q),q}}{E+\hbar w^{+}_{\lambda+q}-\hbar w_{q}}\cdot N_{q}$$

$$+\sum_{q} \frac{A^{2}_{(\lambda+q),q}}{E-\hbar w^{-}_{\lambda+q}-\hbar w_{q}}\cdot N_{q} + \sum_{q} \frac{A^{2}_{\lambda q}(N_{q}-n^{\beta}_{\lambda-q})}{E-\hbar w^{-}_{\lambda-q}+\hbar w_{q}}.\Big]^{-1}$$

Therefore

$$\frac{1}{2\pi}\langle\langle \beta_{\lambda};\beta^{+}_{\lambda}\rangle\rangle^{-1} = (E-\hbar w^{-}_{\lambda} - \sum\nolimits_{m}^{\beta} E(\lambda)) \qquad (3.94)$$

$$\langle\langle b_{q};b^{+}_{q}\rangle\rangle = \frac{1}{2\pi}\Big[E-\hbar w_{q} - \sum_{\lambda} \frac{A^{2}_{\lambda q}}{E-\hbar w^{+}_{\lambda}+\hbar w_{\lambda-q}}(n^{\alpha}_{\lambda-q}-n^{\alpha}_{\lambda})$$

$$+\sum_{\lambda} \frac{B^{2}_{\lambda q}}{E-\hbar w^{+}_{\lambda}-\hbar w_{\lambda-q}}(1+n^{\beta}_{\lambda-q}+n^{\alpha}_{\lambda})$$

$$+\sum_{\lambda} \frac{A^{2}_{\lambda q}(n^{\beta}_{\lambda-q})}{E+\hbar w^{-}_{\lambda}-\hbar w^{+}_{\lambda-q}}\Big]^{-1}$$

Therefore

$$\frac{1}{2\pi} \langle\langle b_q ; b_q^+ \rangle\rangle^{-1} = [E - \hbar w_q - \sum_p E(q)] \qquad (3.95)$$

Here $\sum_m^\alpha E(\lambda)$, $\sum_m^\beta E(\lambda)$ and $\sum_p E(q)$ are self-energy parts of the two magnons modes and phonons respectively. Their principal parts give the renormalisation energies whereas the imaginary parts are related to the life-times of corresponding quasi-particles.

For evaluating the renormalised magnon and phonons energies from the above expressions, we replace E's occurring in the integrands by their unperturbed values.

However, a very important result follows from the expressions (3.93) and (3.94). We note that the two magnon modes are no longer degenerate even in the absence of the external magnetic field. The relative shift will now become a function of temperature.

We shall evaluate the quantities under the long wave-length approximation in the low temperature region. For this purpose, explicit forms of the coupling coefficients will be required.

We evaluate these coefficients under the long wave approximation ignoring the anisotropy energy i.e. we assume

$$\tanh 2\theta_{\underline{\lambda}} = -\gamma_{\underline{\lambda}} \qquad (3.96)$$

Thus we have

$$\begin{matrix} |A_{\underline{\lambda}\underline{q}}|^2 \\ |B_{\underline{\lambda}\underline{q}}|^2 \end{matrix} = D^2 a^4 [\{\underline{\lambda}^6 - \underline{\lambda}^4 |\underline{\lambda} - \underline{q}|^2 - \underline{\lambda}^2 |\underline{\lambda} - \underline{q}|^4 + |\underline{\lambda} - \underline{q}|^6$$

$$+ \underline{q}^4 |\underline{\lambda} - \underline{q}|^2 + \underline{q}^4 \underline{\lambda}^2 - 2\underline{q}^2 |\underline{\lambda} - \underline{q}|^4 - 2\underline{q}^2 \underline{\lambda}^4 + 4\underline{q}^2 \underline{\lambda}^2 |\underline{\lambda} - \underline{q}|^2\}$$

$$/ \; 4\underline{\lambda} |\underline{\lambda} - \underline{q}|$$

$$\pm (\underline{q}^4 - \underline{\lambda}^4 - |\underline{\lambda} - \underline{q}|^4 + 2\underline{\lambda}^2 |\underline{\lambda} - \underline{q}|^2) / 2]$$

$$\ldots\ldots (3.97)$$

where

$$D^2 = \frac{16S^2}{N} \left(\frac{\hbar}{2w_q M} \right) [\, {}^{e}J(R_h)]^2 \qquad (3.98)$$

This is still a fairly complicated form. One can simplify this further fo some specific situations such as $|\underline{\lambda}| \ll |\underline{q}|$, $|\underline{\lambda}| \gg |\underline{q}|$ and $|\underline{\lambda}| \sim |\underline{q}|$

Under these conditions, the coupling coefficients assume the simplified forms (See Appendix E)

for $|\underline{\lambda}| \gg |\underline{q}|$ or $|\underline{\lambda}| \ll |\underline{q}|$

$$A^2_{\underline{\lambda}\underline{q}} = B^2_{\underline{\lambda}\underline{q}} = D^2 \frac{a^4 \underline{\lambda}\, \underline{q}^4}{|\underline{\lambda} - \underline{q}|} (1 + \cos^2 \theta_{\underline{\lambda}\underline{q}}) \qquad (3.99)$$

and for $|\lambda| \sim |q|$

$$A^2_{\underline{\lambda}\underline{q}} = B^2_{\underline{\lambda}\underline{q}} = \frac{D^2 a^4 |\underline{\lambda} - \underline{q}| \underline{q}^4}{\underline{\lambda}} \qquad (3.100)$$

After integration over q as per details shown in appendices E and F, the real part of the self-energy for Magnon modes comes out to be

$$\mathrm{Re} \sum_{m}^{\alpha} E(\lambda) = (A_{\alpha} - B_{\alpha} T^{4})\lambda \tag{3.101}$$

$$\mathrm{Re} \sum_{m}^{\beta} E(\lambda) = (A_{\beta} - B_{\beta} T^{4})\lambda \tag{3.102}$$

where A_{α}, A_{β} are the zero point phonon contributions and B_{α} and B_{β} arise from thermal phonons. Their explicit forms are

$$A_{\alpha} = \frac{S^{2} [{}^{e}J(R_{h})]^{2} \hbar a^{2}}{M v_{S} k_{B}} \left[\frac{7\Theta_{D} - 9\Theta_{a}}{2(\Theta_{D}^{2} - \Theta_{a}^{2})} \right] \tag{3.103}$$

$$B_{\alpha} = \frac{4 S^{2} [{}^{e}J(R_{h})]^{2} \hbar a^{2}}{3\pi^{2} M v_{s} k_{B}} \frac{(9\Theta_{a}^{5} + 8\Theta_{D}^{5} + 13\Theta_{a}^{4}\Theta_{D})}{\Theta_{a}^{4}\Theta_{D}^{4} (\Theta_{D}^{2} - \Theta_{a}^{2})} \Gamma(4)$$

.... (3.104)

$$A_{\beta} = 0 \tag{3.105}$$

$$B_{\beta} = \frac{4 S^{2} [{}^{e}J(R_{h})]^{2} \hbar a^{2}}{\pi^{2} M v_{S} k_{B}} \frac{[8\Theta_{D}^{5} - 8\Theta_{D}^{3}\Theta_{a}^{2} + 7\Theta_{D}\Theta_{a}^{4} - \Theta_{a}^{5}]}{\Theta_{a}^{4} \Theta_{D}^{4} (\Theta_{D}^{2} - \Theta_{a}^{2})} \Gamma(4)$$

.... (3.106)

where $\Theta_{D} = \hbar v_{s} / k_{B} a$, the Debye temperature

and

$$\theta_a = \frac{2SJ\sqrt{2z}}{k_B}$$

and $\Gamma(4)$ is the gamma function.

Thus the renormalised energies of the magnon modes are

$$E_\alpha = 2SJ\sqrt{2z}\ \lambda a + g\mu_B H + (A_\alpha - B_\alpha T^4)\lambda \qquad (3.107)$$

$$E_\beta = 2SJ\sqrt{2z}\ \lambda a - g\mu_B H - B_\beta T^4 \lambda \qquad (3.108)$$

It is seen from the above two equations that even in the absence of external magnetic field ($H = 0$) the two branches are no longer degenerate.

The splitting is:

$$\Delta E_{\alpha\beta} = E_\alpha - E_\beta = (A_\alpha - [B_\alpha - B_\beta]\ T^4)\lambda \qquad (3.109)$$

where

$$B_\alpha - B_\beta = \frac{4S^2[{}^eJ(R_h)]^2 ha^2}{3\pi^2 M v_s k_B} \times \frac{8(\theta_D^5 - 3\theta_D^3\theta_a^2 - \theta_D\theta_a^4 - \theta_a^5)}{\theta_a^4\,\theta_D^4\,(\theta_D^2 - \theta_a^2)}$$

$$\ldots\ldots (3.110)$$

The shift measured as a fraction of the unperturbed magnon energy in the absence of magnetic field is

$$\frac{\Delta E_{\alpha\beta}}{E} = \frac{(A_\alpha - [\ B_\alpha - B_\beta\ ]\ T^4)}{2SJ\sqrt{2z}\ \ a} \tag{3.111}$$

The zero-field contribution to this splitting is:

$$\frac{\Delta E_{\alpha\beta}(T=0)}{E} = \frac{A_\alpha}{2SJ\sqrt{2z}\ .\ a}$$

$$= \frac{S[\,{}^{e}J(R_h)]^2\hbar a}{2J\sqrt{2z}\ k_B M v_s} \times \frac{7\theta_D - 9\theta_a}{2(\theta_D^2 - \theta_a^2)} \tag{3.112}$$

In the same manner, we obtain the renormalised phonon energy, The form of the coupling coefficients, being the same as indicated in (3.99) and (3.100). After integration over magnon wave vectors as shown in appendix H we have

$$\mathrm{Re} \sum_p E(q) = \frac{8}{3} \frac{\hbar\, S^2[\,{}^{e}J(R_h)]^2\ a^4}{M v_s k_B\ \theta_a} .\ q^3$$

$$= A_p q^3 \tag{3.113}$$

Thus the renormalised phonon energy becomes

$$E_p = \hbar v_s q\ \left(1 + \frac{A_p}{\hbar v_s}\ q^2 \right) \tag{3.114}$$

The correction comes in a higher order term involving the wave vector. Also upto this order, the correction is temperature independent.

As in the case of ferromagnets, the phonon renormalisation due to magnon-phonon interaction is weaker than the magnon renormalisation.

The life times of the specific magnon and phonon modes are calculated from the formula

$$\frac{1}{\tau_i} = -\frac{2}{\hbar}\,\mathrm{Im}\sum\nolimits_i (\omega + i0^+) \qquad (3.115)$$

In particular

$$\frac{1}{\tau_{m}(^{\alpha}_{\lambda})} = \frac{2\pi}{\hbar}\sum_{\underline{q}} A^2_{(\underline{\lambda}+\underline{q}),\underline{q}}(1+N_{\underline{q}})\,\delta(E-\hbar\omega^+_{\underline{\lambda}+\underline{q}} + \hbar\omega_{\underline{q}})$$

$$+ A^2_{\underline{\lambda}\underline{q}}(1+n^{\alpha}_{\underline{\lambda}-\underline{q}} - N_{\underline{q}})\,\delta(E - \hbar\omega_{\underline{\lambda}-\underline{q}} - \hbar\omega_{\underline{q}})$$

$$+ B^2_{\underline{\lambda}\underline{q}}(n^{\beta}_{\underline{\lambda}-\underline{q}} - N_{\underline{q}})\,\delta(E + \hbar\omega^-_{\underline{\lambda}-\underline{q}} - \hbar\omega_{\underline{q}})$$

...... (3.116)

$$\frac{1}{\tau_{m}(^{\beta})} = \frac{2\pi}{\hbar}\sum_{\underline{q}} B^2_{(\underline{\lambda}+\underline{q})\underline{q}}\cdot N_{\underline{q}}\,\delta(E+\hbar\omega^+_{\underline{\lambda}+\underline{q}} - \hbar\omega_{\underline{q}})$$

$$+ A^2_{(\underline{\lambda}+\underline{q}).\underline{q}}\cdot N_{\underline{q}}\,\delta(E - \hbar\omega^-_{\underline{\lambda}+\underline{q}} - \hbar\omega_{\underline{q}})$$

$$+ A^2_{\underline{\lambda}\underline{q}}(N_{\underline{q}} - n^{\beta}_{\underline{\lambda}-\underline{q}})\,\delta(E-\hbar\omega_{\underline{\lambda}-\underline{q}} + \hbar\omega_{\underline{q}})$$

...... (3.117)

and

$$\frac{1}{\tau_p} = \frac{2\pi}{\hbar} \sum_{\underline{\lambda}} \Big[- A^2_{\underline{\lambda}\underline{q}} (n^{\alpha}_{\underline{\lambda}-\underline{q}} - n^{\alpha}_{\underline{\lambda}})\, \delta\, (E - \hbar w^{+}_{\underline{\lambda}} + \hbar w_{\underline{\lambda}-\underline{q}})$$
$$+ B^2_{\underline{\lambda}\underline{q}} (1 + n^{\beta}_{\underline{\lambda}-\underline{q}} + n^{\alpha}_{\underline{\lambda}})\, \delta\, (E - \hbar w^{+}_{\underline{\lambda}} - \hbar w_{\underline{\lambda}-\underline{q}})$$
$$+ A^2_{\underline{\lambda}\underline{q}}\, n^{\beta}_{\underline{\lambda}-\underline{q}}\, \delta\, (E + \hbar w^{-}_{\underline{\lambda}} - \hbar w^{+}_{\underline{\lambda}-\underline{q}}) \Big] \qquad (3.118)$$

Converting the summation into integration and evaluating the integrals under the long wavelength and low temperature approximation, as shown in detail in appendix H, we have

$$\frac{1}{\tau_{m_\alpha}} = \frac{S^2[{}^eJ(R_h)]^2}{\pi v_s M K_B \Theta_a} \Big[\frac{2\pi^5}{15} + \frac{8T^4}{\Theta_D^4} (1 + e^{-E_m\alpha/k_BT}) \Big] \qquad (3.119)$$

$$\frac{1}{\tau_{m_\beta}} = \frac{16}{\pi} \frac{S^2 a[{}^eJ(R_h)]^2}{M v_s k_B \Theta_a} \Big[\frac{T^4}{\Theta_D^4} (1 + e^{-E_m\beta/k_BT}) \Big] \qquad (3.120)$$

and

$$\frac{1}{\tau_p} = \frac{16}{\pi} \frac{a^3 S^3[{}^eJ(R_h)]^2}{M v_s k_B \Theta_a} \Big[\frac{\pi^3}{3} + \frac{3T^3}{\Theta_a^3} (1 + 2e^{-E_\beta/k_BT}) \Big] q^2 \qquad \ldots\ldots (3.121)$$

CHAPTER - 4

DISCUSSION AND CONCLUDING REMARKS

In chapters 2 and 3, the renormalisation energies of magnon and phonon modes have been calculated for ferro- and antiferromagnetic systems owing to magnon-phonon interaction. The magnon-magnon and phonon-phonon interaction effects have, however, been neglected.

The results show that the renormalisation effects on phonon modes due to magnon-phonon interaction is quite feeble. However the magnon modes are influenced appreciably.

In the ferromagnetic case, the renormalised magnon energy has the form

$$\tilde{A}_{\lambda} = D(T)\,\lambda^{2} \tag{4.1}$$

with

$$D(T) = D_{o}\,(\,1 - bT^{3/2}\,) \tag{4.2}$$

The temperature variation of the spin-wave coupling parameters D(T) as given in (4.2) is in close accord with the experimental results on thin films of cobalt and nickel as obtained by Phillips and Rosenberg.

In order to compare our parameters with their experimental results, we estimate the quantity b which is given by

$$b = \frac{32}{5\pi} \frac{[S\ {}^{e}J(R_{\underline{h}})]^2\ a^2}{M v_s^2 k_B \theta_c} \left(\frac{T}{\theta_c} \right)^{3/2} \Gamma\left(\frac{3}{2}\right)$$

From an earlier estimation,[9] we have the values

$^{e}J(R_{\underline{h}}) \sim 10^{-6}$ dynes

$\theta_c(\mathrm{Ni}) \sim 630^{\circ}\mathrm{K}$

$a = 3.5 \times 10^{-8}$ cms

$v_s = 5 \times 10^{5}$ cms/sec.

$S = 0.5$

$M = 50 \times 10^{-24}$ gms

Making use of these values, the magnitude of b is

$$b \simeq 2 \times 10^{-5} \left({}^{\circ}\mathrm{K} \right)^{-3/2} \qquad (4.4)$$

This is fairly close to the value obtained from the spin wave resonance as well as from the second order term in magnetisation.

The percentage change in the spin wave coupling parameter $[D_o - D(T)]/D_o \times 100$ from 0° to $300^{\circ}\mathrm{K}$ is found to be of the order of 10 %. This correction also is of the right order as compared to the experimental values of Phillips and Rosenberg[39] on nickel films.

This suggests that phonon-magnon renormalisation effects make the coupling parameter D temperature dependent in the right manner. Also as the derivation is based on the localised spin model, this calculation would thus vindicate the Heisenberg-Bloch model of ferromagnetism in nickel.

Let us now consider the renormalisation effects in a suitable antiferromagnet e.g. MnF_2. The various values of parameters required are[10]

$$S = 2.5$$

$$^{e}J(\underline{R_h}) = 10^{-7} \text{ dynes}$$

$$\Theta_D = 250^{\circ}K, \quad \Theta_a = 30^{\circ}K.$$

$$v_s = 5 \times 10^{5} \text{ cms/sec.}$$

$$a = 2.57 \times 10^{-8} \text{ cm.}$$

$$M = 50 \times 10^{-24} \text{ gms.}$$

A_α, B_α and B_β respectively denote the zero-point phonon contribution for α-magnons, and thermal phonon contributions to α and β magnons, as defined by equations (3.103), (3.104) and (3.106). Their estimated values are as under:

$$A_{\alpha} = 0.14 \times 10^{-24} \text{ erg} \times \text{cm}$$

$$B_{\alpha} = 6.0 \times 10^{-32} \text{ erg.} ({}^{o}K)^{4} \times \text{cm}$$

$$B_{\beta} = 18 \times 10^{-32} \text{ erg.} ({}^{o}K)^{4} \times \text{cm.}$$

Furthermore the zero-point contribution to percentage splitting in the absence of external magnetic field $\Delta E_{\alpha\beta}/E(T=0) \times 100$ comes out of the order of 1%. The temperature dependent correction (Eqn. 3.110) at 50°K is of the order of 0.8×10^{-24} erg. x cm. and is additive to zero-point contribution.

It is therefore reasonable to conclude that two magnon modes are no longer degenerate and it is desirable to look for splitting carefully. That they are not degenerate is not surprising in that the two modes are dynamically different. The splitting caused by magnon-phonon interaction can be explained by the following argument. In the second order, the spin phonon effects after elimination of phonon coordinates lead to effective spin-spin interaction. Thus each sub-lattice sees an effective magnetic field. Inasmuch as each sub-lattice interacts diffeently, the two modes get split up.

CHAPTER - 5

Mathematical Appendices

APPENDIX - A

Magnon self-energy for ferromagnets under low temperature approximation.

By equation (2.51), the Self-Energy for ferromagnetic magnons is given by the expression

$$\sum_m (E_\lambda) = \sum_q \frac{|\phi_{\lambda q}|^2 (-n_{\underline{\lambda}-\underline{q}} + N_{\underline{q}})}{E - A_{\underline{\lambda}-\underline{q}} + \hbar w_{\underline{q}}} +$$

$$+ \sum_q \frac{|\phi_{\underline{\lambda}\underline{q}}|^2 (1+N_{\underline{q}} + n_{\underline{\lambda}-\underline{q}})}{E - A_{\underline{\lambda}-\underline{q}} - \hbar w_{\underline{q}}} \qquad (5.1)$$

Now by Eqn. (2.53)

$$|\phi_{\lambda q}|^2 = \frac{128}{N} \cdot \frac{\hbar}{M v_s q} [S\, {}^eJ(R)]^2 [\lambda^2 q^2 a^4 \cos^2\theta_{\lambda q}] \qquad (5.2)$$

Assuming $\lambda \ll q$, the expression for self energy in (5.1) reduces to:

$$\sum_m (E_\lambda) = \sum_q \frac{128}{N} \frac{\hbar [S\, {}^eJ(R)]^2 . a^4}{M v_s} .$$

$$\cdot \frac{\lambda q^2 \cos^2 \theta_{\lambda q}}{(\hbar v_s q - 2JSq^2 a^2)} (e^{-JSq^2a^2/k_BT}) \qquad \ldots . (5.3)$$

Transforming summation into integration, we have,

$$\sum_m (E_\lambda) = \frac{128}{N} \cdot \frac{\hbar [S\, {}^{e}J(R)]^2\, a^4}{M v_s} \times \frac{V}{8\pi^3}$$

$$\times\ 2\pi \int_0^{q_{max}} \frac{q^2\, e^{-2JSq^2a^2/k_BT}}{(\hbar v_s - 2JSq\, a^2)}\, dq$$

$$\times \int_0^{\pi} \cos^2\theta_{\lambda q}\, \sin\theta_{\lambda q} \cdot d\theta_{\lambda q}. \qquad (5.4)$$

Taking the unperturbed value of the phonon energy in the denominator and noting that $\frac{2JSq_{max}^2 a^2}{k_BT} \gg 1$, the real part of the self-energy,

$$\sum_m (E_\lambda) = -\frac{32}{3\pi^2}\, \frac{[S\, {}^{e}J(R)]^2}{M v_s} \times a^4 \left(\frac{k_BT}{2JS}\right)^{3/2} \Gamma\left(\frac{3}{2}\right) \lambda^2$$

$$\ldots\ldots (5.5)$$

APPENDIX - B

Phonon self-energy for ferromagnets under low temperature approximation.

By equation (2.52), the self-energy of the phonons is given by the expression,

$$\sum_p(w_q) = \sum_{\underline{\lambda}} \frac{|\phi_{\underline{\lambda q}}|^2 (n_{\underline{\lambda}-\underline{q}} - n_{\underline{\lambda}})}{E - A_{\underline{\lambda}} + A_{\underline{\lambda}-\underline{q}}} \tag{5.6}$$

Substituting the value of $|\phi_{\underline{\lambda q}}|^2$ and noting that $\underline{q} \ll \underline{\lambda}$, the expression becomes

$$\sum_p(w_q) = \frac{128}{N} \frac{\hbar[S\,{}^eJ(R)]^2 a^4}{Mv_s} \times \frac{2JSq^2a^2}{k_BT}$$

$$\sum_{\lambda} \lambda^2 \cos^2\theta_{\lambda q}\; e^{-2JS\lambda^2a^2/k_BT} \tag{5.7}$$

Converting summation into integration,

$$\sum_p(w_q) = \frac{128}{N} \frac{\hbar[S\,{}^eJ(R)]^2 a^4}{Mv_s} \frac{2JSq^2a^2}{k_BT} \frac{Na^3}{8\pi^3} \times$$

$$\times\, 2\pi \int_0^{\lambda_{max}} \lambda^4\, e^{-2JS\lambda^2a^2/k_BT}\, d\lambda$$

$$\times \int_0^{\pi} \cos^2\theta_{\lambda q} \cdot \sin\theta_{\lambda q} \cdot d\theta_{\lambda q} \tag{5.8}$$

Noting that

$$\frac{2JS\lambda_{max}^{2} a^{2}}{k_B T} \gg 1,$$

the phonon self-energy is given by

$$\sum_{p}(w_q) = \frac{16}{3\pi^2}\left[\frac{k_B T}{2JS}\right]^{3/2} \frac{[S\,{}^{e}J(R)]^2 a^4}{M v_s^2} \cdot q^2 \, \Gamma(\tfrac{5}{2}) \qquad \text{.............} \quad (5.9)$$

APPENDIX - C

Life-time for ferromagnetic magnons

By equation (2.59), the life-time of magnons is given by

$$\frac{1}{\tau_m(\lambda)} = -\frac{2}{\hbar}\,\mathrm{Im} \sum_m E(\lambda) \tag{5.10}$$

This reduces to

$$\frac{1}{\tau_m(\lambda)} = \frac{2\pi}{\hbar} \sum_q |\phi_{\lambda q}|^2 \; (N_q - n_{\lambda - q})\, \delta(E - A_{\lambda - q} + \hbar w_q)$$

$$+ (1 + N_q + n_{\lambda - q}) \cdot \delta(E + A_{\lambda - q} - \hbar w_q) \tag{5.11}$$

Substituting the values of $|\phi_{\lambda q}|^2$, $A_{\lambda - q}$, $\hbar w_q$ and taking the unperturbed value of E, the expression becomes

$$\frac{1}{\tau_m(\lambda)} = \sum_q \frac{\pi}{N} \times \frac{64}{Mv_s} \left[S\; {}^{e}J(R) \right]^2 a^4 . q\lambda^2 \cos^2 \theta_{\lambda q} \times$$

$$\times\; \delta[\hbar v_s q - 2JSq^2 a^2] \times [1 + 2e^{-2JSq^2a^2/k_BT}]$$

$$\ldots\ldots \quad (5.12)$$

Transforming the sum into the integral we have,

$$\frac{1}{\tau_m(\lambda)} = \frac{\pi x 64}{NMv_s}\left[S\, {}^{e}J(R)\right]^2 a^4 . \lambda^2 . \frac{V}{8\pi^3}$$

$$x\ 2\pi \int_0^{\pi} \sin\theta_{\lambda q} \cos^2\theta_{\lambda q} . d\theta_{\lambda q} \quad x$$

$$x \int_0^{q_{max}} \Big\{ q\, dq \left(1 + 2\, e^{-2JSq^2a^2/k_BT} \right) x$$

$$x\ \delta\left(\hbar v_s q - 2 JSq^2a^2 \right)\Big\} \quad . \qquad (5.13)$$

On integration, we obtain

$$\frac{1}{\tau_m}(\lambda) = \frac{64}{3\pi} . \frac{\hbar\left[S\, {}^{e}J(R)\right]^2 . k_B . \theta_D}{M . \theta_c^3 . k_B^3}\left[1+2e^{-\theta_D^2/\theta_c T}\right] a^2 \lambda^2$$

...... (5.14)

where

$$\theta_D = \hbar v_s / k_B a \quad \text{and} \quad \theta_c = 2JS/k_B \qquad (5.15)$$

APPENDIX - D

Life-time of phonons for ferromagnetic case

The life-time of phonons is given by the expression

$$\frac{1}{\tau_p} = \frac{2\pi}{\hbar} \sum |\phi_{\lambda q}|^2 [n_{\lambda - q} - n_{\lambda}] \delta(E - A_{\lambda} + A_{\lambda - q}) \qquad (5.16)$$

Substituting the values of $|\phi_{\lambda q}|^2$, A_{λ}, $A_{\lambda-q}$, $n_{\lambda-q}$ and n_{λ} and taking the unperturbed value of E, the above expression becomes

$$\frac{1}{\tau_p} = \sum \frac{2\pi}{\hbar} \times \frac{256}{N} \times \frac{h[S\ {}^{e}J(R)]^2 a^4}{2Mv_s} \times \lambda^2 q \cos^2\theta_{\lambda q} \times$$

$$\times \left[e^{2JS(\lambda^2 a^2 + q^2 a^2 - 2\lambda q a^2 \cos\theta_{\lambda q})/k_B T} - e^{-2JS\lambda^2 a^2/k_B T} \right]$$

$$\times \delta[\hbar v_s q + 2JSq^2 a^2 - 2JS\lambda q a^2 \cos\theta_{\lambda q}] \qquad (5.17)$$

Converting summation into integration, we have

$$\frac{1}{\tau_p} = \frac{2\pi}{\hbar} \times \frac{256}{N} \times \frac{\hbar[S\ {}^{e}J(R)]^2 a^4}{2Mv_s}$$

$$\times \frac{V}{8\pi^3} \iint 2\pi \sin\theta_{\lambda q} . \lambda^2 d\lambda . d\theta_{\lambda q} . \lambda^2 q \cos^2\theta_{\lambda q} \times e^{-2JS\lambda^2 a^2/k_B T}$$

$$\left[e^{-2JS(q^2 a^2 - 2\lambda q a^2 \cos\theta_{\lambda q})/k_B T} - 1 \right] \times$$

$$\times \delta[\hbar v_s q + 2JSq^2 a^2 - 2JSq\lambda a^2 \cos\theta_{\lambda q}] \qquad (5.18)$$

On integration between proper limits we find:

$$\frac{1}{\tau_p} = \frac{32}{\pi} \times \frac{\hbar [S\ {}^{e}J(R)]^2 . k_B T}{M.(k_B\,\theta_D)\,(k_B\,\theta_c)^2} \times$$

$$\times \left[\frac{\theta_D}{\theta_c} + qa \right]^2 \left[1 - e^{-\frac{\theta_c}{T}\left(\frac{\theta_D}{\theta_c} + qa\right)^2} \right]$$

$$\times \left[e^{\frac{\theta_c}{T}\left\{ qa + \frac{\theta_D}{\theta_c} - \left(\frac{\theta_D}{\theta_c}\right)^2 \right\}} - 1 \right] \qquad (5.19)$$

where

$$\theta_D = \hbar v_s / k_B a \quad \text{and} \quad \theta_c = 2JS/k_B \qquad (5.20)$$

APPENDIX - B

Real part of the self-energy for antiferromagnetic α - magnons

By equation, (3.97), the coupling coefficient

$$\begin{matrix} A_{\lambda q}{}^{2} \\ B_{\lambda q}{}^{2} \end{matrix} = \Big[\underline{\lambda}^6 - \underline{\lambda}^4 |\underline{\lambda}-\underline{q}|^2 - \underline{\lambda}^2 |\underline{\lambda}-\underline{q}|^4 + |\underline{\lambda}-\underline{q}|^6 + \underline{q}^4 |\underline{\lambda}-\underline{q}|^2$$

$$+ \underline{q}^4 \underline{\lambda}^2 - 2\underline{q}^2 |\underline{\lambda}-\underline{q}|^4 - 2\underline{q}^2 \underline{\lambda}^4 + 4\underline{q}^2 \underline{\lambda}^2 |\underline{\lambda}-\underline{q}|^2 \; / \; 4\,\underline{\lambda}(\underline{\lambda}-\underline{q})$$

$$\pm (\underline{q}^4 - \underline{\lambda}^4 - |\underline{\lambda}-\underline{q}|^4 + 2\underline{\lambda}^2 |\underline{\lambda}-\underline{q}|^2) \; / \; 2 \Big] \times D^2 a^4 \qquad \text{.......... (5.21)}$$

where

$$D^2 = \frac{16S^2}{N} \left(\frac{\hbar}{2w_q M} \right) \left[{}^{e}J(R_h) \right]^2$$

For calculating the magnon self energy, the above coefficient can be approximated to a simplified form by taking the smallest power of wave vector $\underline{\lambda}$ in the above expression. The simplified form of this coefficient is linear in λ and has the following form

$$\begin{matrix} A_{q}{}^{2} \\ B_{q}{}^{2} \end{matrix} = \frac{D^2 a^4 \, . \, \lambda \, q^4}{\underline{\lambda} - \underline{q}} \left(1 + \cos^2 \theta_q \right) \qquad (5.22)$$

The value of $\begin{matrix} A_{(\lambda+q)q}{}^{2} \\ B_{(\lambda+q)q}{}^{2} \end{matrix}$ is the value of the coupling coefficient when $\underline{\lambda}$ is comparable to $\underline{q}$ and can be obtained by taking the smallest power in λ in the expression (5.15) modified by replacing λ by λ +q whenever it occurs. Thus the value of

$$\left.\begin{matrix} A_{(\lambda+q)q}^{\;2} \\ B_{(\lambda+q)q}^{\;2} \end{matrix}\right\} = \frac{D^2 a^2 \lambda\, q^4}{\underline{\lambda} + \underline{q}} \qquad (5.23)$$

Now, the self-energy of α-magnons is given by the expression (Eqn. 3.93):

$$\sum\nolimits_m^{\alpha} E(\lambda) = \sum_q \frac{A_{(\lambda+q)q}^{\;2}}{E-\hbar w^{+}_{\underline{\lambda}+\underline{q}} + \hbar w_{\underline{q}}} (1 + N_q)$$

$$+ \sum_q \frac{A_{\lambda q}^{\;2}}{E-\hbar w_{\lambda-q} - \hbar w_q} (1 + n^{\alpha}_{\lambda-q} - N_q)$$

$$+ \sum_q \frac{B_{\lambda q}^{\;2}}{E + \hbar w^{-}_{\underline{\lambda}-\underline{q}} - \hbar w_{\underline{q}}} (n^{\beta}_{\underline{\lambda}-\underline{q}} - N_q) \qquad \ldots\ldots\ldots\ldots \quad (5.24)$$

In order to evaluate the above self-energy we choose new co-ordinates, $x_1 = \lambda a$, x_2 = qa and $x_{12} = |\underline{\lambda} - \underline{q}|\, a$.

Now the total volume element $d\underline{x}_1 \cdot d\underline{x}_2$ is given in new coordinates as

$$d\underline{x}_1 \cdot d\underline{x}_2 = 8\pi^2\, x_1\, dx_1\, x_2\, dx_2\, x_{12}\, dx_{12}$$

Thus

$$dx_2 \Big|_{x_2=\text{constant}} = \int_{x_2'} dx_1 \, dx' \, \delta(x_2 - x_2')$$

$$= \frac{2\pi x_2 \, x_{12} \, dx_2 \cdot dx_{12}}{x_1} \qquad (5.25)$$

To facilitate summation put $(-q)$ for (q) in the first term of the expression (5.24) and take

$$N_q = e^{-\hbar v_s q/k_B t} = e^{-\hbar v_s/a \, . \, x_2/k_B T}$$

$$N_{\lambda-q}^{\alpha,\beta} = e^{-2JS/V\overline{2z}(\lambda-q)a \, / \, k_B T} = e^{-2JS/V\overline{2z} \, . x_{12}/k_B T}$$

Also substitute the unperturbed value for E in the expression (5.24)

With the above assumptions and converting the various sums into integrals by the expression (5.19), the real part of the self-energy for α-magnons is found to be

$$(A_\alpha - B_\alpha T^4) \qquad (5.26)$$

where

$$A_\alpha = \frac{S^2 [\, {}^e J(R_h)]^2 \hbar a^2}{M v_s K_B} \quad \frac{(7\theta_D - 9\theta_a)}{2(\theta_D^2 - \theta_a^2)}$$

and

$$B_\alpha = \frac{4S^2[\ ^eJ(R_h)]^2\hbar a^2}{3\pi^2 M v_s k_B} \left[\frac{9\theta_a^5 + 8\theta_D^5 + 13\theta_a^4\,\theta_D}{\theta_a^4\,\theta_D^4\,(\theta_D^2 - \theta_a^2\,)} \right] \Gamma(4)$$

with

$$\theta_D = \frac{\hbar v_s}{k_B a}$$

and

$$\theta_a = \frac{2SJ\sqrt{2z}}{k_B}$$

and

$\Gamma(4)$ is the Gamma function.

APPENDIX - F

Real part of the self-energy for antiferromagnetic β - magnons

By the equation (3.94), the expression for the real part of the self-energy for β-magnons is:

$$\sum_{q} \frac{\left|B_{\underline{\lambda}+\underline{q},\,\underline{q}}\right|^2}{E + \hbar w^{+}_{\underline{\lambda}+\underline{q}} - \hbar w_{\underline{q}}} \cdot N_{\underline{q}} + \sum_{q} \frac{\left|A_{(\underline{\lambda}+\underline{q},\underline{q})}\right|^2}{E - \hbar w^{-}_{\underline{\lambda}+\underline{q}} - \hbar w_{\underline{q}}}$$

$$+ \sum_{q} \frac{\left|A^2_{\underline{\lambda}\underline{q}}\right| (N_{\underline{q}} - n^{\beta}_{\underline{\lambda}-\underline{q}})}{E - \hbar w^{-}_{\underline{\lambda}-\underline{q}} + \hbar w_{\underline{q}}} \qquad (5.27)$$

Using the values of coupling coefficients deduced in equations (5.21) and (5.22) and using the volume element defined by equation (5.25) and generally following the method of integration for α-magnons as in Appendix E, the real part of the self-energy for β-magnons is:

$$- \frac{4S^2[\,{}^{e}J(R_h)]^2\, \hbar a^2 T^4}{\pi^2\, M v_s k_B} \left[\frac{8\Theta_D^5 - 8\Theta_D^3\Theta_a^2 + 7\Theta_D\Theta_a^4 + \Theta_a^5}{\Theta_a^4\, \Theta_D^4\, (\,\Theta_D^2 - \Theta_a^2\,)} \right] \Gamma(4)$$

...... (5.28)

where the symbols Θ_D and Θ_a have the same meanings as defined in Appendix E.

The result can be expressed as :

$$\text{Re} \sum\nolimits^{\beta}_{m} E(\lambda) = (A_\beta - B_\beta\, T^4)\,\lambda$$

where

$$A_\beta = 0$$

and

$$B_\beta = \frac{4 S^2 [\,{}^{e}J(R_h)]^2 \hbar a^2}{\pi^2 M v_s k_B} \; \frac{[8\Theta_D^5 - 8\Theta_D^3\Theta_a^3 + 7\Theta_E\Theta_a^4 - \Theta_a^5]}{\Theta_a^4\,\Theta_D^4\,(\Theta_D^2 - \Theta_a^2)}\,\Gamma(4)$$

.... (5.29)

APPENDIX - G

Real part of phonon self-energy for antiferromagnetic case

By Equation (3.95), the real part of the phonon self-energy for antiferromagnetic case is

$$- \sum_{\underline{\lambda}} \frac{|A_{\underline{\lambda}\underline{q}}|^2}{E - \hbar w^{+}_{\underline{\lambda}} + \hbar w_{\underline{\lambda}-\underline{q}}} \left(n^{\alpha}_{\underline{\lambda}-\underline{q}} - n^{\alpha}_{\underline{\lambda}} \right)$$

$$+ \sum_{\underline{\lambda}} \frac{|B_{\underline{\lambda}\underline{q}}|^2}{E - \hbar w^{+}_{\underline{\lambda}} - \hbar w_{\underline{q}}} \left(1 + n^{\beta}_{\underline{\lambda}-\underline{q}} + n^{\alpha}_{\underline{\lambda}} \right)$$

$$+ \sum_{\underline{\lambda}} \frac{|A_{\underline{\lambda}\underline{q}}|^2}{E + \hbar w^{-}_{\underline{\lambda}} - \hbar w^{+}_{\underline{\lambda}+\underline{q}}} \cdot n^{\beta}_{\underline{\lambda}-\underline{q}} \qquad (5.30)$$

The value of the coupling coefficients $|A_{\lambda q}|^2$ by $|B_{\lambda q}|^2$ taking the smallest power of q in the expression (3.97) works out to be

$$\frac{D^2 a^4 \cdot \underline{\lambda}\, \underline{q}^4}{|\underline{\lambda}-\underline{q}|} \left(1 + \cos^2 \theta_{\underline{\lambda}\underline{q}} \right) \qquad (5.31)$$

For transforming the sum into integral, it is convenient to choose new coordinates

$$\underline{\lambda} a = x_1$$

$$\underline{q} a = x_2$$

and

$$(\underline{\lambda}-\underline{q}) a = x_{12}$$

and to use the volume element

$$\frac{2\pi x_1 \cdot x_{12} \cdot dx_1 \cdot dx_{12}}{x_2}$$, on a basis similar to

the element used in Equation (5.19)

$$n^{\alpha}_{\lambda-q} = e^{-2JS\sqrt{2z}.x_{12}/k_BT} \quad \text{etc.} \tag{5.32}$$

and E can be replaced by the unperturbed value of the energy.

Using the above technique we find

$$\mathrm{Re}\sum_p E(q) = \frac{8}{3}\frac{\hbar S^2[\,{}^{e}J(R_h)]^2 a^4}{Mv_s.\ k_B.\ \Theta_a} \cdot q^3 \tag{5.33}$$

$$= A_p \cdot q^3$$

where

$$A_p = \frac{8}{3}\frac{\hbar S^2[\,{}^{e}J(R_h)]^2 a^4}{Mv_s \cdot k_B.\ \Theta_a} \tag{5.34}$$

APPENDIX - H

(i) Life-Time for α-magnons in antiferromagnetic case:

By equation (3.116) the life-time $_m$ for α-magnons is given by

$$\frac{1}{\tau_m(\alpha_\lambda)} = \frac{2\pi}{\hbar}\sum_q |A^2_{(\lambda+q)q}|(1+N_q)\,\delta(E-\hbar w^+_{\lambda+q} + \hbar w_q)$$

$$+|A^2_{\lambda q}|(1+n^\alpha_{\lambda-q}-N_q)\,\delta(E-\hbar w_{\lambda-q} - \hbar w_q)$$

$$+|B^2_{\lambda q}|(n^\beta_{\lambda-q}-N_q)\,\delta(E+\hbar w^-_{\lambda-q} - \hbar w_q) \qquad (5.35)$$

For evaluating the above, the values of coupling coefficients $|A_{(\lambda+q)q}|^2$, $|A_{\lambda q}|^2$ and $|B_{\lambda q}|^2$ as defined by Equations (5.16) and (5.17) are to be used in conjunction with the values of N_q, $n^\alpha_{\lambda-q}$ under low-temperature approximation and the unperturbed value of the energy E.

Using new co-ordinates x_1, x_2 and x_{12} and the volume element defined by Equation (5.25) the sum can be converted into integral with several terms.

After integration,

$$\frac{1}{\tau_m(\alpha_\lambda)} = \frac{S^2[\,{}^eJ(R_h)]^2}{\pi v_s M k_B\,\theta_a}\left[\frac{2\pi^5}{15} + \frac{8T^4}{\theta_D^4}\left(1+e^{-E_{m_\alpha}/k_BT}\right)\right. \qquad \ldots\ldots (5.36)$$

where

$$\theta_D = \frac{\hbar v_s}{k_B a} \;, \quad \theta_a = \frac{2JS\sqrt{2z}}{k_B} \quad \text{and} \quad E_{m_\alpha} = 2JS\sqrt{2z}.\lambda a$$

(ii) Life-time of β-magnons for antiferromagnetic case

The life-time for β-magnons is to be calculated from Equation (3.117)

$$\frac{1}{\tau_m(\beta_\lambda)} = \frac{2\pi}{\hbar} \sum_{\underline{q}} |B^2_{(\underline{\lambda}+\underline{q}),\underline{q}}| N_{\underline{q}} . \delta\,(E + \hbar w^+_{\underline{\lambda}+\underline{q}} - \hbar w_{\underline{q}})$$

$$+ |A^2_{(\underline{\lambda}+\underline{q}),\underline{q}}| N_{\underline{q}} . \delta(E - \hbar w^-_{\underline{\lambda}+\underline{q}} - \hbar w_{\underline{q}})$$

$$+ |A^2_{\underline{\lambda}\underline{q}}| (N_{\underline{q}} - n^\beta_{\underline{\lambda}-\underline{q}})\; \delta(E - \hbar w^-_{\underline{\lambda}-\underline{q}} + \hbar w_{\underline{q}}) \qquad (5.37)$$

Proceeding in the same way as for α-magnons, we find

$$\frac{1}{\tau_{m\beta}} = \frac{16}{\pi} \; \frac{S^2 a[\,{}^eJ(R_h)]^2}{M v_s k_B\, \theta_a} \; [\; \frac{T^4}{\theta_D^4} \,(\, 1 + e^{-E_{m\beta}/k_B T}\,)] \qquad \text{..... (5.38)}$$

(iii) Life-time of phonons for anti-ferromagnetic case.

The life-time of phonons is given by the equation (3.118) as:

$$\frac{1}{\tau_p} = \frac{2\pi}{\hbar}\sum_{\underline{\lambda}} \Big[|-A_{\underline{\lambda}\underline{q}}|^2 (n^{\alpha}_{\underline{\lambda}-\underline{q}} - n^{\alpha}_{\underline{\lambda}})\, \delta(E - \hbar w^{+}_{\underline{\lambda}} + \hbar w_{\underline{\lambda}-\underline{q}})$$

$$+ |B_{\underline{\lambda}\underline{q}}|^2 (1 + n^{\beta}_{\underline{\lambda}-\underline{q}} + n^{\alpha}_{\underline{\lambda}}) \cdot \delta(E - \hbar w^{+}_{\underline{\lambda}} - \hbar w_{\underline{\lambda}-\underline{q}})$$

$$+ |A_{\underline{\lambda}\underline{q}}|^2\, n^{\beta}_{\underline{\lambda}-\underline{q}}\, \delta(E + \hbar w^{-}_{\underline{\lambda}} - \hbar w^{+}_{\underline{\lambda}-\underline{q}}) \Big] \qquad (5.39)$$

For evaluating the above, the values of coupling coefficients defined by Equation (5.31) is to be used in conjunction with the values of n^{α} and E using the unperturbed energy values.

It is also convenient to use the co-ordinates x_1, x_2 and x_{12} for λa, qa and $(\underline{\lambda} \pm \underline{q})a$ and to use the volume element

$$\frac{2\pi\ x_1 \cdot x_{12} \cdot dx_1\ dx_{12}}{x_2}$$ for converting the sum

into integral between suitable limits.

On integration, we find

$$\frac{1}{\tau_p} = \frac{16}{\pi} \frac{a^3 S^2 [\ {}^{e}J(R_h)]^2}{M v_s \cdot k_B \cdot \Theta_a} \left[\frac{\pi^3}{3} + \frac{3T^3}{\Theta_a^3} (1 + 2e^{-E_p/k_B T}) \right] q^2 \qquad \ldots\ldots (5.40)$$

Where E_p is unperturbed phonon energy and other symbols have the meanings defined earlier.

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Sharda Prasad Saxena.
(SHARDA PRASAD SAXENA)