INTERACTIONS INVOLVING ELEMENTARY EXCITATIONS IN FERROMAGNETIC SEMICONDUCTORS

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A C K N O W L E D G E M E N T

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(S. S. Shah)

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No part of this thesis has been submitted for a degree or diploma or other academic award. The literature concerning the problems investigated has been surveyed, and all the necessary references are given in the thesis. The present work has been clearly indicated at appropriate places separately. In accordance with the usual practice, due acknowledgement has been made wherever the work presented is based on the results of other worker.

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C O N T E N T S

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S Y M B O L S

1. k, λ and q are the electron, magnon and phonon wave vectors (They are not underlined)

2. ϵ_k, E^m_λ and E^p_q are the single particle electron, magnon and phonon energies with wave vectors k, λ and q respectively.

3. A_q, $J_m(k,\lambda)$ and $\phi_{\lambda q}$ are the electron-phonon, electron-magnon and phonon-magnon coupling coefficients.

4. $\bar{A}_q$ and $\bar{\phi}_{\lambda q}$ are the constant parts of the above coupling coefficients (wave vector independent part).

5. T_r relaxation time.

6. E_f Fermi energy.

7. k_β Boltzmanns constant.

8. I dispersion constant for magnons.

C H A P T E R - I

C H A P T E R - I

GENERAL INTRODUCTION

Owing to their numerous applications, the study of magnetic materials is of great interest and importance. Quantum theoretical study of magnetic materials is essential as the microscopic mechanism of various physical properties, origin of magnetism and the role played by interactions among the various elementary excitations on the transport and other properties are not well understood. Here a new interaction which has not been taken into account so far, has been considered and its effects on various thermodynamic and magnetic properties are studied in great detail. The motivation behind this work is to show how the interactions among various elementary excitations play a pronounced role in the physical properties of magnetic systems.

Among the magnetically coupled systems, we shall be interested in those types of crystalline solids which contain an assembly of atoms or ions with permanent magnetic moment. It is generally agreed that exchange forces between electrons provided the coupling between elementary magnets. Exchange interaction represents a characteristic quantum effect having no analog in classical mechanics. Exchange interaction in magnetic materials can be studied for insulators (including semiconductors) and metals. In insulators and semiconductors,

the spins and magnetic moments whose alignment leads to magnetic effects are localisable, so that, they can be described by a spin Hamiltonian which contains spin operators and exchange terms of Heisenberg[1] type. Frenkel[2] and Heisenberg were the first to show that in ferromagnetism, the main role is played by exchange interaction between the electrons. In Heisenberg model, a certain number of unpaired electrons are assumed to be present in each atoms, regularly spaced in the crystal. It was shown in Heisenberg's theory that an exchange effect leading to a strong coupling between the electronic spins would be caused by ordinary coulomb interactions among electrons when proper cognizance of the exclusion principle is taken. Two methods were proposed in 1929 to deal with it. Determinantal method was proposed by Slater[3-8] and spin operator method was proposed by Dirac[9-10] The spin operator method which has been used widely by Van Vleck[11-15] and his school for magnetic problems, will be discussed here. Dirac showed that the spin dependent energy for interacting electrons can be written as[16-17]

$$H_{ex} = - \sum_{ij} J_{ij} \underline{S}_i \cdot \underline{S}_j \qquad (1.1)$$

where $\underline{S}_i$ is the spin operator for the electron in the i^{th} orbital. Let ψ_a and ψ_b be the localised atomic orbitals satisfying the equations.

$$\left(\frac{p_i^2}{2m} - \frac{Ze^2}{r_{ia}} \right) \psi_a(i) = E_a \psi_a(i) \qquad (1.2)$$

$$\left(\frac{p_j^2}{2m} - \frac{Ze^2}{r_{jb}} \right) \psi_b(i) = E_b \psi_b(j) \qquad (1.3)$$

where i, j represent position of electrons and a, b that of the nucleus and $p_i^2/2m$ and $-Ze^2/r_{ia}$ are the K.E. and potential energy of i^{th} electron. In terms of these wave functions, the exchange integral (J) in general case, is given by

$$J_{ij} = \langle ab \mid e^2/r_{ij} \mid ba \rangle - 2S_{ab} \langle a \mid V \mid b \rangle \qquad (1.4)$$

Here

$$V = \frac{-Ze^2}{r_{ib}} - \frac{Ze^2}{r_{ja}}$$

and

$$S_{ab} = \langle a \mid b \rangle = \int \psi_a^* \psi_b \, d\tau$$

In the case of orthogonal orbitals, the overlap integral $S_{ab} = 0$ and only the first term in (1.4) survives. This term is always positive being the self energy of overlap charge $e\psi_a^*(i)\psi_b(j)$. This favours ferromagnetism (The triplet state). However if the orbitals are non-orthogonal[19] the sign of J_{ij} would depend on whether the first or second term dominates and when the second term dominates the antiferromagnetic coupling (the singlet state) is favoured. The theory of direct exchange interaction between two paramagnetic atoms formed the basis of much theoretical work on ferro and

anti-ferromagnetism. For ferromagnetic insulators (semi-conductors), which contain transition elements also, the indirect exchange phenomena i.e. s-d (f) exchange deserves special mention.

We have seen that ferromagnetism demands positive 'J'. A controversy arose about this conjecture because the usual sign of J is negative, as is necessary in order that a chemical bond may be formed. The reasoning of Heisenberg "that J can change sign from positive to negative for sufficiently large value of principal quantum number, was not satisfactory. This argument could not explain the absence of ferromagnetism in second and third transition period, which according to his reasoning should be stronger than the first transition period. Slater[19] postulated that as the degree of overlap of 'd' shells decreased from a large to a small value, J changes from negative to positive. Zener[20-22] in 1951, pointed out that, the necessity for a positive J disappeared if cognizance is taken of the new interaction which had been neglected till then. When magnetic crystal have conduction electrons (mainly due to 'S' electrons), the role of exchange interaction between the localised magnetic electrons (say the 'd' electrons) and the conduction electrons may become important. In this case the corresponding spin Hamiltonian[23-26] is written as

$$H_{ex}(s\text{-}d) = -\,1/N \sum_{k,k'l} J(k,k')\, e^{i(k-k').R_l} \quad X$$

$$[(c^{+}_{k'\uparrow} c_{k\uparrow} - c^{+}_{k'\downarrow} c_{k\downarrow})\; S^{z}_{l} + c^{+}_{k'\uparrow} c_{k\downarrow} S^{-}_{l} + c^{+}_{k'\downarrow} c_{k\uparrow} S^{+}_{l}\,] \qquad \ldots\ldots (1.5)$$

where we have carried out summation over spins and used the relationship between spin operators and fermion operators for localised electrons. J(k,k') is the exchange integral.

The s-d exchange interaction can give rise to a coupling between the two 'd' atoms via the polarisation of conduction electrons. This coupling appears in second order of s-d interaction after the conduction electron spin terms are eliminated. The effective interaction term is given by

$$H_{eff} = \sum_{lm} I(R_{lm})\; \underline{S}_l \cdot \underline{S}_m \qquad (1.6)$$

where

$$I(R_{lm}) = -\,\frac{9\pi}{2}\,\frac{N_s^2}{N^2}\,\frac{J_o^2}{E_f}\; F(2k_o R_{lm})$$

and

$$F(x) = \frac{x\cos x - \sin x}{x^4}$$

where

$$x = (\,2\,k_o\,R_{lm}\,)$$

and $J_o = J_{(k,k')}$. As the exact dependence of J on (k,k')

is not known, we assume it as constant, J_o. Its order of magnitude is given by Izyumov[27]. N_s is total number of s electrons and E_f the Fermi energy. k_o is the propagation vector at the fermi surface. The sign of interaction term depends on the function $F(2k_o R_{lm})$. Rudermann and Kittel[28] also have studied similar interaction. It is to be noted that the magnitude of this interaction decreases as the third power of distance between magnetic atoms. Unlike Heisenberg exchange interaction, the indirect coupling has a long range character. It appears therefore, the direct interaction may not be dominant process in the ferro or antiferromagnetic ordering (particularly in metals). Furthermore, the indirect process is of particular significance in the magnetic ordering in dilute alloys, rare earth metals and some magnetic compounds.

Elementary Excitations in Solids

Most of the physical properties of solids are governed by interactions involving elementary excitations. Their use to describe the complicated interrelated motion has turned out to be an extraordinarily useful device in contemporary physics, and it is this view of solid which we wish to adopt in proceeding further.

We can regard a solid as a collection of essentially independent elementary excitations, provided the excitation possesses a well defined energy. An excitation with momentum $\mathbf{k}$ will have energy of the form

$$E_k = E_k - i\gamma_k \qquad (1.7)$$

where γ_k is the imaginary part of the energy and is inversely proportional to the life time of the excitation. An excitation is well defined if it is long lived, which requires

$$\gamma_k \ll E_k .$$

This requirement confines us generally to very low temperatures and to the phenomena which involve comparatively low frequencies and long wave-lengths. Even when above criterion is not satisfied, the physical process may be usefully described by the excitations involved but it is necessary to take into account the fact that the excitations possess a finite life time.

It should be emphasized that the elementary excitations are the result of collective interaction of the particles of the system, and therefore pertain to the system as a whole and not to its separate particles. In particular, the number of elementary excitation is certainly not the same as the total number of particles in the system. Also the statistics of the elementary excitations do not have to be same as the statistics of the particles making up the system. All energy spectra can be classified into two broad categories spectra of Bose type and the spectra of Fermi-Dirac type[29]. In the first type, the excitations have integral valued spins obeying Bose statistics, while in the second case the excitations have half-integral spins and obey Fermi-Dirac statistics.

Earlier we have briefly described the various exchange mechanisms which lead to a definite ordering in the magnetic materials (ferro, antiferro and ferrimagnetic). The definite ordering is realised at absolute zero of temperature and the magnetic moments are aligned in a definite pattern, (ferro, anti-ferro, or ferrimagnetic). One can speak of this state of affairs as the ground state of the whole crystal. If the temperature is slightly increased, excitation in the spin system as well as the lattice excitation will be produced over the ground state. Thus if we know the energy spectrum of the elementary excitations of a particular system, we can have a fairly clear idea of its physical behaviour. Thus we are mainly going to consider the two excitations i.e. spin and lattice and the their combined effect on the physical properties of the system under investigation.

Magnons.

Bloch[30] first introduced the concept of spin wave. The low lying energy states of the systems coupled by exchange interactions are wavelike. In the ground state of the spin system, which will be realised at $0^{o}K$, each spin has the maximum allowable value S_i^z, namely S. A spin wave may be described as a sinusoidal disturbance of spin system with amplitude at each lattice site proportional to $S - S_i^z$. A completely new technique was suggested by Holstein and Primakoff (H.P.)[31] to include the spin wave interactions which was followed by a

rigorous and satisfying treatment by Dyson[32-33], HP[31] successfully defined a set of coordinates which have the appearance of the spin wave amplitude and which describe accurately the quantum state of the system.

The spin Hamiltonian (with ferromagnetic exchange integral)

$$H = -\sum_{i,j} J_{ij}\, \underline{S}_i \cdot \underline{S}_j \qquad (1.8)$$

involves three components S_{ix}, S_{iy} and S_{iz} of each spin $\underline{S}_i$. Also

$$\underline{S}_i \cdot \underline{S}_i = S(S+1) \qquad (1.9)$$

Let us define two new operators

$$\underline{S}_i^+ = \underline{S}_{ix} + i\,\underline{S}_{iy} = (2\underline{S})^{1/2}\left(1 - \frac{a_i^+ . a_i}{2\underline{S}}\right)$$

$$\underline{S}_i^- = \underline{S}_{ix} - i\,\underline{S}_{iy} = (2S)^{1/2}\, a\left(1 - \frac{a_i^+ . a_i}{2S}\right) \qquad (1.10)$$

and $n_i = S - S_i^z$ where $n_i = a_i^+ a_i$ represents deviation of S^z from the maximum value S.

In general the operators a_i^+ and a_i do not obey Bose-Einstein statistics. In the ground state, all the spins of the d-shells have the same direction i.e. $S_i^z = S$. At low temperatures these deviations are small and we can take

$$\langle a_i^+ a_i \rangle / S = \left\langle \frac{n_i}{S} \right\rangle \ll 1 \qquad (1.11)$$

when above equation is satisfied, the operators a_i^+ and a_i will be having exactly Boson character i.e.

$$[\, a_i, a_j^+ \,] = \delta_{ij}$$

This approximation is called spin wave approximation.

Taking into account (equation 1.11) we get

$$a_i = \frac{S_i^+}{(2S)^{1/2}}, \qquad a_i^+ = \frac{S_i^-}{(2S)^{1/2}} \qquad (1.12)$$

By using the above relations we get,

$$H = - \sum_{i,j} J_{ij} \left[S_i^z S_j^z + \frac{1}{2} (S_i^+ S_j^- + S_i^- S_j^+) \right] \qquad (1.13)$$

$$\sum_{i,j} J_{ij} \left[S^2 + S(a_i a_j^+ + a_i^+ a_j - a_i^+ a_i - a_j^+ a_j) \right] \qquad (1.14)$$

We have dropped a term of the type $a_i^+ a_i a_j^+ a_j$.

By introducing fourier transforms to the reciprocal space

$$a_i = \frac{1}{\sqrt{N}} \sum_{\lambda} a_\lambda \, e^{i\lambda \cdot R_i}$$

$$a_i^+ = \frac{1}{\sqrt{N}} \sum a_\lambda^+ e^{-i\lambda \cdot R_j} \qquad (1.15)$$

The operators a_i and a_i^+ satisfy the commutation relations

$$[\ a_\lambda, a_{\lambda'}^+\ ] = \delta_{\lambda\lambda'},$$

$$[\ a_\lambda, a_{\lambda'}\ ] = [\ a_\lambda^+, a_{\lambda'}^+\ ] = 0$$

Using (1.15) and the fact that J_{ij} depends only on $\underline{h}=\underline{R}_i-\underline{R}_j$, we get

$$H = H_o + \sum_\lambda \left[\sum_{\underline{h}} 2SJ(\underline{h})(1-e^{-i\lambda\underline{h}}) \right] a_\lambda^+ a_\lambda \qquad 1.16$$

where H_o is independent of the operators a_λs. But $H-H_o$ is the Hamiltonian of a set of single harmonic oscillators, one for each wave vector, having energy spectrum

$$E = n_\lambda \sum_{\underline{h}} 2SJ(\underline{h})\ (1-e^{-i\lambda\underline{h}}) \qquad 1.17$$

Thus a_λ and a_λ^+ are annihilation and creation operators for the spin waves. The energy in the λ^{th} spin waves mode is quantised in units

$$\hbar w_\lambda = \sum_{\underline{h}} 2SJ(\underline{h})(1-e^{-i\lambda\underline{h}})$$

Such an elementary excitation is named as magnon[34-39] in analogy with photons and phonons.

For simple cubic lattice with nearest neighbour interaction we have

$$\left.\begin{aligned} \hbar w_\lambda &= 2SJ\lambda^2 a^2 \\ E_\lambda^m &= I\lambda^2 \end{aligned}\right\} \qquad (1.18)$$

where $I = 2SJa^2$, the dispersion constant.

Phonons

We have introduced the concept of collective oscillations in magnetically ordered systems i.e. spin waves (magnons). However, the ions in the lattice are never quiescent and the collective motion of the ions which are coupled by inter-ionic forces give rise to a very important thermodynamic effects. Their interactions with other entities such as electrons, spins (and hence spin waves) would be equally important.

The oldest concept of elementary excitations came in connection with lattice waves. Any local disturbance due to the strong coupling between atoms, travels like a wave. These lattice waves are characterised by propagation vectors and velocity of propagation. In quantum-mechanical description of lattice waves we visualise the crystal in certain quantum state of excitations. The units of these excitations are referred to as phonons characterised by a definite energy and direction of propagation.

We will briefly describe the subject of lattice interaction, as it has been described rigorously in many standard works[40-46]. Let us consider a system of N ion cores spaced

regularly in a three dimensional net work. The lattice Hamiltonian is given by

$$H_l = \sum_l \frac{P_l^2}{2m} + \sum_{l \neq l'} V(R_l - R_{l'}) \qquad (1.19)$$

R_l, $R_{l'}$, refer to vector coordinates of atoms l and l' at arbitrary positions. P_l denotes the momentum, referred to the equilibrium positions

$$R_l = \underline{R}_l^0 + \delta\underline{R}_l$$

Taking adiabatic approximation[41] into account, we can expand the potential energy in powers of the nuclear displacement from equilibrium configuration and if we break off this expansion after the second power (this approximation is called harmonic approximation), we have

$$H = \sum_l \frac{P_l^2}{2M} + \frac{1}{2} \sum_{l \neq l'} V(R_l^0 - R_{l'}^0) + \sum_{l,l'} \frac{A_{ll'}}{2} \delta R_l \, \delta R_{l'}, \qquad \ldots \quad (1.20)$$

where

$$A_{ll'} = \frac{\partial V(R_l - R_{l'})}{\partial R_l \, \partial R_{l'}} \qquad l \neq l'$$

The constant V can be neglected and

$$\partial V / \partial R_l = 0 \quad \text{in equilibrium.}$$

Then the Hamiltonian takes the form

$$H_L = \frac{1}{2}\sum \frac{P_l P_l}{m} + \frac{1}{2}\sum_{l,l'} \delta R_l \cdot A_{ll'} \cdot \delta R_{l'} \qquad (1.21)$$

introducing the following normal coordinate transformation

$$\delta R_l = \frac{1}{\sqrt{NM}} \sum_{q,p} Q_{qp} \epsilon_{pq} e^{-iq R_l^o}$$

$$P_l = \sqrt{\frac{M}{N}} \sum_{q,p} P_{qp} \epsilon_{pq} e^{iq.R_l^o}$$

ϵ_{qp} are the eigenvectors of the dynamical matrix

$$\sum_{ll'} A_{ll'} e^{-iq.(R_l^o - R_{l'}^o)}$$

Using these transformations we can write

$$H_L = \frac{1}{2}\left(\sum P^+_{qp} P_{qp} + w^2_{qp} Q^+_{pq} Q_{pq} \right)$$

where

$$w^2_{qp} = \frac{A_{qp}}{M} \qquad \text{and}$$

$$A_{qp} = \epsilon_{qp} \left[\sum_{ll'} A_{ll'} e^{-iq.(R_l^o - R_{l'}^o)} \right] \epsilon_{qp}$$

Here we have restricted ourself to the first Brillouin zone and used the result

$$\sum_{l} e^{i(q-q').R_l^o} = N \delta_{q,q'} .$$

We now introduce the Boson creation and annihilation operators b_{qp}^{+} and b_{qp} through the relations

$$Q_{qp} = i \left(\frac{\hbar}{2w_{qp}}\right)^{1/2} (b_{qp}^{+} - b_{-qp})$$

$$P_{qp} = \left(\frac{\hbar w_{qp}}{2}\right)^{1/2} (b_{qp} + b_{-qp}^{+}) .$$

It can be easily shown that they obey the appropriate commutation relations

$$[\, b_{qp} , b_{q',p'}^{+}] = \delta_{qq'} \delta_{pp'}$$

$$[\, b_{qp}, b_{q'p'}] = [\, b_{qp}^{+}, b_{q'p'}^{+}] = 0$$

Finally making use of the second quantisation representation[47-51] we get

$$H_L = \sum \hbar w_{qp} (b_{qp}^{+} b_{qp} + 1/2) \qquad (1.22)$$

with $b_{qp}^{+} b_{qp} = n_{qp}$ the occupation number operator, which has only positive integers for its eigenvalues. It is evident from the above expression (1.22) that the vibrational field of the lattice vibration can be regarded as a system of non-interacting Boson particles called phonons, each having an energy $\hbar w_{qp}$.

For a cubic crystal with the wave vector $\underline{q}$ of the lattice wave along a symmetry axis, the polarisation direction may be fixed by symmetry arguments. This will lead to the notion of longitudinal and transverse modes of vibrations. The longitudinal mode is a essentially a compressional wave and the transverse mode is a shear wave. The presence of more than one atom in the unit cell leads to the different modes of vibrations known as acoustic and optical modes[52-53]. In the acoustic mode the atoms of the unit cell move in the same direction and in the optical mode they move in opposite direction. The frequency of the acoustic mode tends to zero as the wave vector q_a of the acoustic phonon tends to zero. In the Debye model the acoustic mode frequency is proportional to q_a and can be put up into the form $w_{qa} = v_c q_a$ where v_c is the velocity of sound. In the optical mode the frequency remains finite as $\underline{q}$ tends to zero.

The eigen function of H_L can be denoted by $|- - - - N_{qp} - - - -\rangle$ and the above operators then have the properties

$$b^{+}_{qp} \left| - - - - N^{p}_{q} \cdots - \right\rangle = (N_{qp}+1)^{1/2} \left| - - - N_{qp}+1 - - \right\rangle$$

$$b_{qp} \left| - - - - N^{p}_{q} - - - - \right\rangle = (N_{qp})^{1/2} \left| - - - N_{qp}-1 \cdots - \right\rangle$$

Survey of interactions among elementary excitations

In a magnetic crystal, at finite temperature both collective modes i.e. lattice waves and spin waves exist simultaneously. We have seen that the corresponding quasi-particles or elementary excitations are phonons and magnons. In the low temperature region, we speak of a magnetic crystal harbouring a cloud of phonons and magnons. Hence we come across three predominant interactions in view of transport properties. The interactions are 1) Electron-phonon interaction, 2) Electron-magnon interaction, and 3) phonon-magnon interactions. The last one will be investigated rigorously in subsequent chapters, as the first two interactions have already been studied in detail by various authors. We briefly review these interactions here.

A) The effect of electron-phonon[54-57] interaction are manifold and determine a large variety of phenomena in metals, semiconductors and insulators e.g. superconductivity, resistivity, lattice polarisation effect etc. These interaction effects have therefore, been studied at length in many solids. Conduction electrons visualise in various ways any deformation of the ideal periodic lattice of positive ion cores. Even the zero-point motion of phonons has its effect on the conduction electron. The consequences of the coupling of electrons and phonons can be summarised as below.

a) The most familiar one is the scattering of electrons from one state K to another K' leading to electrical resistivity. Second one is the absorption of phonons by the electrons. This offers a mechanism for the attenuation of the sound waves, or, in higher order, the thermal resistance of the metal. The electron-phonon interaction also causes the attractive interaction between electrons which is important for superconductivity and results from the virtual emission and absorption of a phonon. Lastly the electron will always carry with it a co-moving cloud of phonons (lattice polarisation field). The composite particle, electron plus phonon field is called polaron. The phonon cloud changes the energy of electron.

The interaction[58-61] of the conduction electrons in a ferromagnet with spin waves is analogous to the polaron problem if we consider electron and spin waves to be a separate subsystems. The Hamiltonian consists of three terms H(electron) - H_{el}, H(spin-wave) - H_{sw} and H_{int}. Here H_{el} is one electron Hamiltonian and the Heisenberg spin Hamiltonian H_{sw} describes the coupling of spin on the neighbouring sites. H_{int} is the effective intra-atomic exchange interaction proportional to the scalar product of the 'd' electron with the total spin at the site and therefore couples the electron to the spin-wave system. In polaron problem, the system can attain lower energy by distortion of the lattice and localisation of the charge carrier. Similar effect has been suggested by Zener for electron interacting with spin wave. As we observe effective mass enhancement of electron due to electron-

phonon interaction, similar enhancement of effective mass of electron will be observed due to electron-magnon-interaction as well. The self-energy of an electron in a ferromagnetic metal due to the virtual emission and absorption of magnons has been calculated. It was found that the mass correction due to electron magnon interaction may be as large as that due to the electron phonon interaction[62-63]. The difference in the two interactions will be as follows. An electron in an arbitrary state is capable of both absorbing and emitting phonons, the transition to the new state having the same spin direction and satisfying the energy-momentum conservation laws. To the contrary exchange interaction between the electron and the spin wave produces only electron transition in which the spin changes the direction but the total spin of the system is conserved. The electron-magnon interaction also leads to a splitting of the energy spectrum of the up and down ($\uparrow$ and $\downarrow$) electrons by some amount. Also the consequences of electron-magnon interactions such as resistivity relaxation, thermal emf etc. are well-known.

In the low temperature region, we can speak of a crystal harbouring a cloud of phonons and magnons which are excitations of lattice and spin systems, respectively. In the harmonic approximation, both of these particles are non-interacting. However, there are processes which do indeed bring about phonon-phonon, magnon-magnon and phonon-magnon interactions. So, when a crystal is not subjected to external perturbations,

all quasi particles find themselves in a state of equilibrium distribution, characterised by one temperature. The equilibrium can be disturbed by either exciting the spin or lattice system, each characterised by a temperature say T_s and T_l. At this stage the interactions of elementary excitations (quasi-particles) with each other leads to the establishment of statistical equilibrium. For this purpose, the process of establishment of equilibrium means process of equalisation of temperature. Such an approach is permissible if the time of establishing of equilibrium with each of the subsystems (phonons and magnons) is considerably shorter than the relaxation between the subsystems. In fact phonon-phonon and magnon-magnon relaxation processes are much faster than the phonon-magnon relaxation processes. Thus understanding of phonon-magnon interaction is essential. Basic microscopic phenomena of phonon-magnon interaction in various magnetic solids was developed in the last few years[67-71]. These processes were successfully applied towards the study of relaxation and transport properties. The precise knowledge of energy spectrum of excitation in magnetically ordered system, will be complete, if we take the effect of phonon-magnon interaction on their energy renormalisation. There are experimental indications[72] that the magnon-phonon interaction terms would influence the energy spectrum of magnon and phonon modes. The renormalisation of magnon modes will be of considerable importance in the studies of magnetisation and other properties.

Conclusions

The purpose of the present work is to investigate the interactions involving electrons, phonons and magnons in a ferromagnetic system. The Hamiltonian of the system is formulated and then appropriate single particle, double time temperature Green functions are set up. The equations of motion of the Green functions are then set up, which involve higher order Green functions. After decoupling at a suitable stage of hierarchy and taking the energy fourier transform, the coupled equations are solved. The self energy parts of various Green functions are obtained. These in turn involve mass and polarisation terms. The life times of the quasi-particles are obtained from the imaginary parts of the Green functions.

A P P E N D I X

The Greens Function Technique[73-76]

One of the important concepts of quantum field theory is that of Green functions, which are convenient for the study of properties of interacting quantised fields. These concepts turn out to be useful in statistical mechanics as well. This method is particularly useful owing to the simplicity in its formulation and interpretation and when combined with spectral representations it provides a powerful tool for attacking various problems. The green functions in quantum mechanics are appropriate generalisation of the concept of correlation functions. They are intimately connected with the evaluation of the observed quantities and they have well-known advantages when equations are formulated and solved.

We establish the notation, the principal definitions and basic equations of this technique in this section for double time temperature dependent green functions used in our work. We define two types of Green functions, the retarded and the advanced Green functions.

$$
\begin{aligned}
G_r(t,t') &= \langle\langle A(t);\ B(t') \rangle\rangle_r \\
&= -i\,\theta(t-t') \left\langle [A(t),B(t')] \right\rangle \\
G_a(t,t') &= \langle\langle A(t);\ B(t') \rangle\rangle_a \\
&= i\,\theta(t-t') \left\langle [(A(t);B(t')] \right\rangle
\end{aligned}
\qquad (1)
$$

where A(t) and B(t') are the time dependent Heisenberg operators given by

$$A(t) = e^{iHt} A e^{-iHt} \quad \text{and} \quad \langle - - - - \rangle$$

indicates an average over grand canonical ensemble hence

$$[A(t');B(t')] = Z^{-1} \text{ trace } e^{-H\beta}[A(t),B(t')] \qquad (2)$$

where $Z = \text{Trace } e^{-H\beta}$ and $H = H' - \mu N$ where H' is the Hamiltonian of the system, μ is the chemical potential and $\beta = (k_\beta T)^{-1}$. k_β is the Boltzman's constant.

$\Theta(t)$ is defined by

$$\Theta(t) = 1 \quad t > 0$$

$$= 0 \quad t < 0$$

The square bracket represents commutator or anticommutator

$$[A,B] = A\,B - nBA$$

where $n = +1$ for Bosons

$n = -1$ for Fermions

We define some useful quantities such as correlation functions F_{AB} and F_{BA} as follows

$$\left.\begin{aligned} F_{BA}(t,t') &= \langle B(t'), A(t) \rangle \\ F_{AB}(t,t') &= \langle A(t), B(t) \rangle \end{aligned}\right\} \qquad (3)$$

We can show that, G_r and G_a can be expressed as linear combinations of the above functions.

If we use the equations of motion for the Heisenberg operators, we can derive the equations of motion for the Green functions. Using

$$\frac{i\,\partial A}{\partial t} = [A, H]_{-1} \quad \text{and}$$

$$\frac{\partial \theta}{\partial t}(t-t') = \frac{\partial \theta}{\partial t}(t'-t) = \partial(t-t')$$

We find for the two Green functions, the same equation of motion i.e.

$$i\frac{\partial}{\partial t}\langle\langle A,B\rangle\rangle = \partial(t-t')\langle[A,B]_{-n}\rangle + \langle\langle[A,H]_{-1}\,;\,B\rangle\rangle \qquad (4)$$

In thermodynamic equilibrium the G's defined, depend on time only through $(t-t')$ i.e.

$$F_{BA}(t-t') = \langle B(t'),A(t)\rangle = F_{BA}(t-t')$$

We write[73-76]

$$F_{BA}(t-t') = \int_{-\infty}^{\infty} J(w)\, e^{-iw(t-t')}\, dw \qquad (5)$$

$$J(w) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_{BA}(t)\, e^{iwt}\, dt \qquad (5')$$

It can also be shown that

$$F_{AB}(t-t') = \int_{-\infty}^{\infty} J(w)\, e^{\beta w}\, e^{-iw(t-t')}\, dw \qquad (6)$$

We can write

$$G(t,t') = \int_{\infty}^{\infty} G(E)\, e^{-iE(t-t')}\, dE \qquad (7)$$

where

$$G_{r,a}(E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G_{r,a}(t-t')\, e^{iE(t-t')}\, d(t-t') \qquad (7')$$

Substituting for $G_r(t-t')$ from equations (1, 3, 5 and 6) we get

$$G_{r,a}(E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} (e^{\beta w} - n)\, J(w)\, \frac{dw}{e - w \pm i\epsilon}$$

In arriving at this result we have used the relation[74]

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} dt\, e^{i(E-w)t}\, \theta(t) = \frac{i}{2\pi}\, \frac{1}{E - w + i\epsilon} \qquad (8)$$

It can be shown that the function $G_r(E)$ can be analytically continued in the upper half plane and $G_a(E)$ in the lower half If a cut is made along the real axis

$$G(E) = G_r(E) \quad \text{Im } E > 0$$

$$= G_a(E) \quad \text{Im } E < 0$$

We, from the definition of J(w) can show that

$$G(w + i\epsilon) - G(w - i\epsilon) = -i\,(e^{\beta w} - n)\,J(w) \qquad (9)$$

In deriving this equation we have used the identity

$$\lim_{\epsilon \to 0,\ > 0} \frac{1}{x \pm i\epsilon} = \frac{P}{x} \mp i\pi\,\delta(x)$$

Hence

$$J(w) = i\,\frac{[\,G(w+i\epsilon) - G(w-i\epsilon)\,]}{e^{w\beta} - 1} \qquad (10)$$

and

$$\Gamma_{BA}(t-t') = \langle B(t'), A(t) \rangle$$

$$= i \int_{-\infty}^{\infty} \frac{G(w+i\epsilon) - G(w-i\epsilon)}{e^{w\beta} - n}\, e^{-iw(t-t')}\, dw \qquad \ldots.(11)$$

Using this relation we can obtain expressions for the distribution functions of the electrons, phonons and magnons and their approximate energy values.

In the present work, the effect of magnon-phonon interaction on the renormalisation of quasi-particle energies is studied. The Hamiltonian for the ferromagnetic system is formulated. Then we introduce double time temperature dependent Green functions for electron, magnon and phonons. A set of coupled equations of motion are derived and solved after suitable decoupling approximations. Also lifetime of each quasi-particle is derived from the relation between relaxation time and the imaginary part of self energy.

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C H A P T E R - II

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FORMULATION OF HAMILTONIAN AND GREEN FUNCTION METHOD FOR FERROMAGNETISM

Here we will develop the theory of phonon-magnon interactions by using the double time temperature dependent Green Functions. We have chosen ferro-magnetic insulators (semiconductors) crystallising in a body centred cubic structure for convenience. The total Hamiltonian for such a system consists of following parts:

$$H = H_e + H_m + H_p + H_{em} + H_{ep} + H_{pm} \qquad (2.1)$$

where H_e is the one electron Hamiltonian and is given in the second quantised representation by

$$H_e = \sum_{k,\sigma} \epsilon_{k\sigma} c^+_{k\sigma} c_{k\sigma} \qquad (2.2)$$

ϵ_k is the single particle electron energy, $\underline{k}$ is the wave vector. c^+_k, c_k are the Fermion creation and annihilation operators, and σ is the spin index. H_m and H_p are the magnon and phonon Hamiltonians with

$$H_m = \sum_{\lambda} E^m_\lambda \left(a^+_\lambda a_\lambda + 1/2 \right) \qquad (2.3)$$

$$H_p = \sum_{q} E^p_q \left(b^+_q b_q + 1/2 \right) \qquad (2.4)$$

where a^+_λ , a_λ and b^+_q , b_q are respectively creation and

annihilation operators for magnons and phonons of wave vector λ and q. They include mode branches. E^m_λ and E^p_q are the magnon and phonon single particle energies. The explicit form of the electron-magnon interaction term is given by[1, 13]

$$H_{em} = -\sum_{k,\lambda} J_m(k,\lambda)\,[c^+_{k-\lambda\uparrow}c_{k\downarrow}\,a^+_\lambda + c^+_{k\downarrow}\,c_{k-\lambda\uparrow}a_\lambda\,]$$

$$+\frac{1}{2}\sum_{kk'} J(k,k')\,\delta(k'-k+\lambda-\lambda')[c^+_{k\uparrow}c_{k'\uparrow}-c^+_{k\downarrow}c_{k'\downarrow}]a^+_\lambda\,a_\lambda \qquad \ldots\ldots(2.5)$$

The role of the second term in equation (2.5) is to make the one particle electron energy spin dependent i.e. $\epsilon_{k\sigma}$ and it modifies the electron energy by amount $S(1\pm/u)J$. Hence we take the energy of the electron as $\epsilon^{\pm}_k = \epsilon_k - S(1 \pm/u)J$. $\pm$ sign represents the up and down spin electrons respectively. Hence we will take H_{em} as

$$H_{em} = -\sum_{k,\lambda} J_m(k,\lambda)[c^+_{k-\lambda\uparrow}c_{k\downarrow}a^+_\lambda + c^+_{k\downarrow}c_{k-\lambda\uparrow}a_\lambda\,] \qquad (2.6)$$

where $J_m(k,\lambda)$ is the s-d exchange coupling coefficient.

The electron-phonon interaction term is given by[13-20]

$$H_{ep} = \sum_{q,k,\sigma} A_q[\,c^+_{k+q,\sigma}\,c_{k,\sigma}b_q + c^+_{k-q,\sigma}c_{k,\sigma}\,b^+_q\,] \qquad (2.7)$$

A_q being the electron phonon coupling coefficient.

Finally the phonon-magnon interaction[21-23] is given as follows:

$$H_{pm} = \sum_{\lambda,q} \phi_{\lambda q} (a^{+}_{\lambda+q}\, a_{\lambda}\, b_{q} + a^{+}_{\lambda+q}\, a_{\lambda}\, b^{+}_{-q}) \qquad (2.8)$$

where $\phi_{\lambda q}$ is the phonon-magnon coupling coefficient.

We now define the double time temperature dependent retarded Green functions which are of interest for the present problem

$$\left.\begin{aligned} G^{\sigma\sigma'}_{kk'}(t-t') &= \langle\langle c_{k,\sigma}(t),\ c^{+}_{k',\sigma'}(t') \rangle\rangle \\ G_{qq'}(t-t) &= \langle\langle b_{q}(t),\ b^{+}_{q'}(t') \rangle\rangle \\ G_{\lambda\lambda'}(t-t') &= \langle\langle a_{\lambda}(t),\ a^{+}_{\lambda'}(t') \rangle\rangle \end{aligned}\right\} \qquad (2.9)$$

As indicated in Appendix to Chapter I (Green function technique), we set up the equations of motion of these Green functions,

$$\begin{aligned} i\frac{\partial}{\partial t} \langle\langle c_{k\sigma};\ c^{+}_{k',\sigma'}(t') \rangle\rangle &= \delta(t-t')\, \delta_{kk'}\, \delta_{\sigma\sigma'} \\ &+ \epsilon_{k\sigma} \langle\langle c_{k,\sigma}(t);\ c^{+}_{k',\sigma'}(t') \rangle\rangle \\ &- \sum_{\lambda} J_{m}(k,\lambda)\, [\, \delta_{\sigma\uparrow} \langle\langle c_{k+\lambda\downarrow} a^{+}_{\lambda}(t);\ c^{+}_{k',\sigma'}(t') \rangle\rangle \\ &+ \delta_{\sigma\downarrow} \langle\langle c_{k-\lambda\uparrow} a_{\lambda}(t),\ c^{+}_{k',\sigma'}(t') \rangle\rangle] \\ &+ \sum_{q} A_{q}\, [\, \langle\langle c_{k+q,\sigma} b^{+}_{q}(t),\ c^{+}_{k',\sigma'}(t') \rangle\rangle \\ &+ \langle\langle c_{k-q,\sigma}\, b_{q}(t);\ c^{+}_{k',\sigma'}(t') \rangle\rangle] \end{aligned} \qquad (2.10)$$

Taking the fourier transform and rearranging the terms we get

$$(w - \epsilon_{k\sigma}) \ll c_{k\sigma}, c^{+}_{k',\sigma'} \gg_w = \frac{1}{2\pi} \delta_{kk'} \delta_{\sigma\sigma'}$$

$$- \sum_{\lambda} J_m(k,\lambda) [\delta_{\sigma\uparrow} \ll c_{k+\lambda\downarrow} a^{+}_{\lambda}, c^{+}_{k',\sigma'} \gg_w$$

$$+ \delta_{\sigma\downarrow} \ll c_{k-\lambda\uparrow} a_{\lambda}; c^{+}_{k',\sigma'} \gg_w]$$

$$+ \sum_{q} A_q [\ll c_{k+q,\sigma} b^{+}_{q}; c^{+}_{k',\sigma'} \gg_w + \ll c_{k-q,\sigma} b_{q}; c^{+}_{k',\sigma'} \gg_w]$$

..... (2.11)

The subscript 'w' after the Green functions represents that we have taken energy fourier transform of the equations (2.10). We will omit the subscript 'w' hereafter.

In analogous way, we obtain following equation for phonon Green function given in equation (2.9)

$$(w - E^{p}_{q}) \ll b_q; b^{+}_{q} \gg = \frac{1}{2\pi} + \sum_{k,\sigma} A_q \ll c^{+}_{k-q,\sigma} c_{k\sigma}; b^{+}_{q} \gg$$

$$+ \sum_{\lambda} \phi_{\lambda q} \ll a^{+}_{\lambda-q} a_{\lambda}; b^{+}_{q} \gg \qquad (2.12)$$

and the equation for magnon Green function will be

$$(w - E^{m}_{\lambda}) \ll a_{\lambda}, a^{+}_{\lambda} \gg = \frac{1}{2\pi} - \sum_{k} J_m(k,\lambda) \ll c^{+}_{k-\lambda\uparrow} c_{k\downarrow}; a^{+}_{\lambda} \gg$$

$$+ \sum_{q} \phi_{\lambda q} [\ll a_{\lambda-q} b_q; a^{+}_{\lambda} \gg + \ll a_{\lambda-q} b^{+}_{-q}; a^{+}_{\lambda} \gg]$$

....(2.13)

These are single particle Green functions i.e. those involving three operators. Also equation (2.11) involves only electron-phonon and electron-magnon coupling terms. To incorporate phonon-magnon interaction effects, we shall be required to solve them for higher order Green functions (i.e. the four higher order Green functions involved in equation 2.11). We get the following equation of motion (after taking the energy fourier transform) for these four higher order Green functions involved in equation 2.11.

$$
\begin{aligned}
&(w - \epsilon_{k+\lambda} + E^{m}_{\lambda}) \ll\!\ll c_{k+\lambda\downarrow} a^{+}_{\lambda} \; ; \; c^{+}_{k',\sigma'} \gg\!\gg \\
&= - \sum_{k_1,\lambda_1} J_m(k_1,\lambda_1) \; [\; \delta_{k_1,k+\lambda} \ll\!\ll c_{k_1-\lambda_1\uparrow} a^{+}_{\lambda} a_{\lambda_1} \; ; \; c^{+}_{k',\sigma'} \gg\!\gg \\
&\quad - \delta_{k_1 k+\lambda} \; \delta_{\lambda\lambda_1} \ll\!\ll c_{k_1-\lambda_1\uparrow} ; c^{+}_{k',\sigma'} \gg\!\gg \\
&\quad - \delta_{\lambda\lambda_1} \ll\!\ll c_{k+\lambda\downarrow} \; c^{+}_{k_1\downarrow} \; c_{k_1-\lambda_1\uparrow} ; \; c^{+}_{k',\sigma'} \gg\!\gg \;] \\
&+ \sum_{q} A_q [\ll\!\ll c_{k+\lambda-q\downarrow} a^{+}_{\lambda} \; b_q \; ; \; c^{+}_{k',\sigma'} \gg\!\gg \\
&\quad + \ll\!\ll c_{k+\lambda+q\downarrow} a^{+}_{\lambda} \; b^{+}_{q} \; ; \; c^{+}_{k',\sigma'} \gg\!\gg \;] \\
&+ \sum_{\lambda_1,q} \phi_{\lambda_1 q} [\; \delta_{\lambda\lambda_1} \ll\!\ll c_{k+\lambda\downarrow} \; a^{+}_{\lambda_1+q} \; b_q \; ; \; c^{+}_{k',\sigma'} \gg\!\gg \\
&\quad \delta_{\lambda\lambda_1} \ll\!\ll c_{k+\lambda\downarrow} \; a^{+}_{\lambda_1+q} \; b^{+}_{q} \; ; \; c^{+}_{k',\sigma'} \gg\!\gg \;] \qquad (2.14)
\end{aligned}
$$

$$(w - \epsilon_{k-\lambda} - E_\lambda^m) \langle\langle c_{k-\lambda\uparrow} a_\lambda ; c^+_{k',\sigma'} \rangle\rangle =$$

$$- \sum_{k_1,\lambda_1} J_m(k_1,\lambda_1)[\delta_{k_1-\lambda_1,k-\lambda} \langle\langle c_{k_1\downarrow} a_\lambda a^+_{\lambda_1} ; c^+_{k',\sigma'} \rangle\rangle$$

$$- \delta_{k_1-\lambda_1,k-\lambda}\, \delta_{\lambda\lambda_1} \langle\langle c_{k_1\downarrow} ; c^+_{k',\sigma'} \rangle\rangle$$

$$+ \delta_{\lambda\lambda_1} \langle\langle c_{k-\lambda\uparrow} c^+_{k_1-\lambda_1\uparrow} c_{k_1\downarrow} ; c^+_{k',\sigma'} \rangle\rangle \;]$$

$$+ \sum_q A_q[\, \langle\langle c_{k-\lambda-q\uparrow} a_\lambda b_q \;;\; c^+_{k',\sigma'} \rangle\rangle$$

$$+ \langle\langle c_{k-\lambda+q\uparrow} a_\lambda b^+_q \;;\; c^+_{k',\sigma'} \rangle\rangle \;]$$

$$+ \sum_{\lambda_1,q_1} \phi_{\lambda_1 q_1}[\, \delta_{\lambda_1+q_1} \langle\langle c_{k-\lambda\uparrow} a_{\lambda_1} b_q ; c^+_{k',\sigma'} \rangle\rangle$$

$$\delta_{\lambda_1+q_1} \langle\langle c_{k-\lambda\uparrow} a_\lambda b^+_{-q} ; c^+_{k',\sigma'} \rangle\rangle \;] \qquad (2.15)$$

$$(w - \epsilon_{k+q} + E_q^p) \langle\langle c_{k+q,\sigma} b^+_{-q} ; c^+_{k',\sigma'} \rangle\rangle$$

$$- \sum_{k_1,\lambda_1} J_m(k_1,\lambda_1)[\, \delta_{k+q,k_1-\lambda_1}\, \delta_{\sigma\uparrow} \langle\langle c_{k_1\downarrow} b^+_q a^+_{\lambda_1} ; c^+_{k',\sigma'} \rangle\rangle$$

$$+ \delta_{k_1,k+q}\, \delta_{\sigma\downarrow} \langle\langle c_{k_1-\lambda_1\uparrow} b^+_q a_\lambda ; c^+_{k',\sigma'} \rangle\rangle \;]$$

$$+\sum_{q_1,k_1} A_{q_1}\Big[\ \delta_{k+q,k_1+q_1}\ \delta_{\sigma-\sigma_1}\ \delta_{qq_1}\ll c_{k_1\sigma_1};\ c^+_{k',\sigma'}\gg$$

$$+\ \delta_{k+q,k_1+q_1}\ \delta_{\sigma\sigma_1}\ll c_{k_1\sigma_1} b^+_q\ b_{q_1}\ ;\ c^+_{k',\sigma'}\gg$$

$$-\ \delta_{qq_1}\ll c_{k+q,-\sigma} c^+_{k_1+q_1\sigma_1}\ c_{k_1\sigma_1}\ ;\ c^+_{k',\sigma'}\gg$$

$$+\ \delta_{k+q_1,k_1-q_1}\ \delta_{\sigma-\sigma_1}\ll c_{k_1\sigma_1} b^+_{q_1}\ b^+_q\ ;\ c^+_{k',\sigma'}\gg\ \Big]$$

$$+\sum_{\lambda_1 q_1}\phi_{\lambda_1 q_1}\ \delta_{q\ q_1}\ll a^+_{\lambda_1+q_1}\ a_{\lambda_1} c_{k+q_1,-\sigma}\ ;\ c^+_{k',\sigma'}\gg \qquad (2.16)$$

and

$$(w-\epsilon_{k-q}-E^p_q)\ll c_{k-q,-\sigma}\ b_q\ ;\ c^+_{k',\sigma'}\gg =$$

$$-\sum_{k_1,\lambda_1} J_m(k_1,\lambda_1)\Big[\ \delta_{k-q,k_1-\lambda_1}\ \delta_{\sigma\uparrow}\ll c_{k_1\downarrow} b_q\ a^+_{\lambda_1}\ ;\ c^+_{k',\sigma'}\gg$$

$$-\ \delta_{k_1,k-q}\ \delta_{\sigma\downarrow}\ll c_{k_1-\lambda_1\uparrow}\ a_\lambda b_q\ ;\ c^+_{k',\sigma'}\gg\ \Big]$$

$$+\sum_{q_1,\lambda_1} A_{q_1}\Big[\ \delta_{k-q,k_1+q_1}\ \delta_{\sigma\sigma_1}\ll c_{k_1\sigma_1}\ b_q\ b_{q_1}\ ;\ c^+_{k',\sigma'}\gg$$

$$+\ \delta_{k-q,k_1-q_1}\ \delta_{\sigma-\sigma_1}\ll c_{k_1\sigma_1}\ b_q\ b^+_{q_1}\ ;\ c^+_{k',\sigma'}\gg$$

$$-\ \delta_{qq_1}\ll c^+_{k_1-q_1\sigma_1} c_{k-q,-\sigma}\ c_{k_1\sigma_1}\ ;\ c^+_{k',\sigma'}\gg\ \Big] \qquad (2.17)$$

We also solve for the higher order Green functions involved in equations (2.11) and (2.12) and we get following equations

$$(w-\epsilon_{k-q}-\epsilon_k)\ \langle\langle c_{k-q,\sigma}\, c_{k\sigma}\,;\, b_q^+\rangle\rangle =$$

$$= -\sum_{k_1,\sigma_1} J_m(k_1,\lambda_1)\,[\,\delta_{k_1,k-q}\langle\langle c^+_{k_1-\lambda_1\uparrow} c_{k\downarrow}\ a^+_\lambda\ ;\ b_q^+\rangle\rangle$$

$$-\ \delta_{k,k_1-\lambda}\langle\langle c^+_{k-q\uparrow}\, c_{k_1\downarrow}\, a^+_\lambda\ ;\ b_q^+\rangle\rangle - \delta_{k_1-\lambda,k-q}\langle\langle c^+_{k_1\downarrow} c_{k_1\uparrow} a_\lambda\,; b_q^+\rangle\rangle$$

$$-\ \delta_{kk_1}\langle\langle\, c^+_{k-q\downarrow}\, c_{k_1-\lambda_1\uparrow}\, a_\lambda\ ;\ b_q^+\rangle\rangle\ ]$$

$$+\sum_{q_1,\sigma_1} A_{q_1}\delta_{k,k_1+q_1}\ \delta_{\sigma\sigma_1}\langle\langle c^+_{k-q,\sigma}\, c_{k_1\sigma_1}\ b_q\ ;\ b_q^+\rangle\rangle$$

$$-\ \delta_{k_1,k-q}\ \delta_{\sigma\sigma_1}\langle\langle c^+_{k_1+q_1,\sigma_1}\ c_{k_1\sigma_1}\ b_q\ ;\ b_q^+\rangle\rangle$$

$$-\ \delta_{k_1,k-q}\ \delta_{\sigma\sigma_1}\langle\langle c^+_{k_1-q_1\sigma_1}\ c_{k\sigma}\ b^+_{q_1}\ ,\ ;\ b_q^+\rangle\rangle$$

$$+\ \delta_{k,k_1-q_1}\ \delta_{\sigma\sigma_1}\langle\langle c^+_{k-q,\sigma}\ c_{k_1\sigma_1}\ b^+_{q_1}\ ;\ b_q^+\rangle\rangle\ ] \qquad (2.18)$$

$$(w-E^m_{\lambda-q}-E^m_\lambda)\ \langle\langle a^+_{\lambda-q}\, a_\lambda\ ;\ b_q^+\rangle\rangle =$$

$$= -\sum_{k,\lambda_1} J_m(k,\lambda_1)\ [\,\delta_{\lambda\lambda_1}\langle\langle c^+_{k-\lambda_1\uparrow}\, c_{k\downarrow}\, a^+_{\lambda-q}\ ;\ b_q^+\rangle\rangle$$

$$-\ \delta_{\lambda_1,\lambda-q_1}\langle\langle c^+{}_{k\downarrow}\, c_{k-\lambda_1\uparrow}\, a_\lambda\ ;\ b_q^+\rangle\rangle\ ]$$

$$+\sum_{\lambda_1,q_1}\phi_{\lambda_1 q_1}\,[\,\delta_{\lambda,\lambda_1+q_1}\langle\langle a^+_{\lambda-q}\ a_{\lambda_1}\ (b_q+b^+_{-q})\ ;\ b_q^+\rangle\rangle$$

$$-\ \delta_{\lambda_1,\lambda-q}\ \langle\langle\, a^+{}_{\lambda_1+q_1}\ a_\lambda(b_q+b^+_{-q})\ ;\ b_q^+\rangle\rangle\ ] \qquad (2.19)$$

$$(w - \epsilon_k + \epsilon_{k-\lambda}) \ll\!\!\ll c^+_{k-\lambda\uparrow} c_{k\uparrow} \; ; \; a^+_\lambda \gg\!\!\gg \;\; =$$

$$- \sum_{k_1 \lambda_1} J_m(k_1 \lambda_1) \; [\; \delta_{k k_1} \ll\!\!\ll c^+_{k-\lambda\uparrow} c_{k_1-\lambda_1} \; a_{\lambda_1} \; ; \; a^+_\lambda \gg\!\!\gg$$

$$- \delta_{k_1-\lambda_1, k-\lambda} \ll\!\!\ll c^+_{k_1\downarrow} c_{k\downarrow} \; a_{\lambda_1} \; ; \; a^+_\lambda \gg\!\!\gg]$$

$$+ \sum_{k_1 q_1} A_{q_1} \; [\; \delta_{k, k_1+q_1} \ll\!\!\ll c^+_{k-\lambda\uparrow} c_{k_1\downarrow} \; b_{q_1} \; ; \; a^+_\lambda \gg\!\!\gg$$

$$- \delta_{k_1, k-\lambda} \ll\!\!\ll c^+_{k_1+q_1\uparrow} c_{k\downarrow} b_{q_1} \; ; \; a^+_\lambda \gg\!\!\gg$$

$$- \delta_{k_1, k-\lambda} \ll\!\!\ll c^+_{k_1-q_1\uparrow} c_{k\downarrow} b^+_{q_1} \; ; \; a^+_\lambda \gg\!\!\gg$$

$$- \delta_{k, k_1-q_1} \ll\!\!\ll c^+_{k-\lambda\uparrow} c_{k_1\downarrow} \; b^+_{q_1} \; ; \; a^+_\lambda \gg\!\!\gg] \qquad (2.20)$$

$$(w - E^p_q - E^m_{\lambda-q}) \; \ll\!\!\ll a_{\lambda-q} \; b_q, \; a^+_\lambda \gg\!\!\gg$$

$$- \sum_{k, \lambda_1} J_m(k, \lambda_1) \; \delta_{\lambda_1, \lambda-q} \ll\!\!\ll c^+_{k-\lambda_1\uparrow} c_{k\downarrow} \; b_q \; ; \; a^+_\lambda \gg\!\!\gg$$

$$+ \sum_{k, q_1} A_{q_1} \; \delta_{q_1, q} \ll\!\!\ll c_{k-q_1\sigma_1} c_{k\sigma} \; a_{\lambda-q} \; ; \; a^+_\lambda \gg\!\!\gg$$

$$+ \sum_{\lambda_1, q_1} \phi_{\lambda_1 q_1} \; [\; \delta_{\lambda-q, \lambda_1+q_1} \ll\!\!\ll b_{q_1} \; b_q \; a_{\lambda_1} \; ; \; a^+_\lambda \gg\!\!\gg$$

$$- \delta_{\lambda-q, \lambda+q_1} \; \delta_{q_1, -q} \ll\!\!\ll a_{\lambda_1} ; \; a^+_\lambda \gg\!\!\gg + \delta_{q_1, -q} \ll\!\!\ll a_{\lambda-q} a^+_{\lambda+q_1} a_{\lambda_1}, a^+_\lambda \gg\!\!\gg$$

$$+ \delta_{\lambda-q, \lambda_1+q_1} \ll\!\!\ll b_q \; b^+_{-q_1} \; a_{\lambda_1} \; ; \; a^+_\lambda \gg\!\!\gg] \qquad (2.21)$$

$$
\begin{aligned}
&(w + E^{p}_{q} - E^{m}_{\lambda-q}) \ll\!\!\ll a_{\lambda-q}\ b^{+}_{-q}\ ;\ a^{+}_{\lambda} \gg\!\!\gg \\
&+ \sum_{k_1\lambda_1} J_m(k_1\lambda_1)[\delta_{\lambda_1,\lambda-q} \ll\!\!\ll b^{+}_{-q}\ c_{k-\lambda_1\uparrow}\ c_{k\downarrow}\ ;\ a^{+}_{\lambda} \gg\!\!\gg] \\
&- \sum_{q_1 k} A_{q_1} [\delta_{-q,q_1} \ll\!\!\ll c^{+}_{k+q_1\sigma_1}\ c_{k\sigma}\ a_{\lambda-q}\ ;\ a^{+}_{\lambda} \gg\!\!\gg] \\
&+ \sum_{\lambda_1 q_1} \phi_{\lambda_1 q_1} [\delta_{\lambda-q,\lambda_1+q_1}\ \delta_{q_1,-q} \ll\!\!\ll a_{\lambda_1}\ ;\ a^{+}_{\lambda} \gg\!\!\gg \\
&\quad + \delta_{\lambda-q,\lambda_1+q_1} \ll\!\!\ll b^{+}_{-q}\ b_{q_1}\ a_{\lambda_1}\ ;\ a^{+}_{\lambda} \gg\!\!\gg \\
&\quad - \delta_{q_1,-q} \ll\!\!\ll a_{\lambda-q}\ a^{+}_{\lambda_1+q_1}\ a_{\lambda_1}\ ;\ a^{+}_{\lambda} \gg\!\!\gg \\
&\quad + \delta_{\lambda-q,\lambda_1+q_1} \ll\!\!\ll b^{+}_{-q_1}\ b^{+}_{-q}\ a_{\lambda_1}\ ;\ a^{+}_{\lambda} \gg\!\!\gg] \qquad (2.22)
\end{aligned}
$$

For evaluating the single particle phonon and magnon Green Functions, the higher order Green functions noted above (cf. equations 2.18 - 2.22) will be adequate, However, for the electron Green functions, we shall require some higher order equations i.e. those involved in equations (2.14) to (2.18). We will work out one of these Green functions in detail i.e. the Green function involved (Equation 2.14 last but one line).

$$(w-\epsilon_{k+\lambda}+E^{m}_{\lambda+q}-E^{p}_{q})\ll\!\!\ll c_{k+\lambda}\ a^{+}_{\lambda+q}\ b_{q};\ c^{+}_{k',\sigma'}\gg$$

$$=\sum_{k_1\lambda_1}J_m(k_1,\lambda_1)\ [\ \delta_{k_1,k+\lambda}\,\delta_{\lambda_1,\lambda+q}\ll\!\!\ll b_q c_{k_1-\lambda_1\uparrow};\ c^{+}_{k',\sigma'}\gg$$

$$+\ \delta_{k_1,k+\lambda}\ll\!\!\ll a^{+}_{\lambda+q}\ a_{\lambda_1}b_q\ c_{k_1-\lambda_1\uparrow};\ c^{+}_{k',\sigma'}\gg$$

$$-\ \delta_{\lambda_1,\lambda+q}\ll\!\!\ll c_{k+\lambda\downarrow}c^{+}_{k_1\downarrow}\ c_{k_1-\lambda_1\sigma}b_q\ ;\ c^{+}_{k',\sigma'}\gg\!\!\gg$$

$$+\sum_{k_1q_1}A_{q_1}\ [\ \delta_{k_1+q_1,k+\lambda}\ \delta_{\sigma\downarrow}\ll\!\!\ll a^{+}_{\lambda+q}b_q b_{q_1}c_{k_1\sigma}\ ;\ c^{+}_{k',\sigma'}\gg$$

$$-\ \delta_{k_1-q_1,k+\lambda}\,\delta_{\sigma\downarrow}\,\delta_{qq_1}\ll\!\!\ll a^{+}_{\lambda+q}\ c_{k_1\sigma}\ ;\ c^{+}_{k',\sigma'}\gg$$

$$+\ \delta_{k_1-q_1,k+\lambda}\,\delta_{\sigma\downarrow}\ll\!\!\ll a^{+}_{\lambda+q}\ b_q\ b^{+}_{q_1}\ c_{k_1\sigma}\ ;\ c^{+}_{k',\sigma'}\gg$$

$$-\ \delta_{qq_1}\ll\!\!\ll a^{+}_{\lambda+q}\ c_{k+\lambda\downarrow}\ c^{+}_{k_1-q_1,\sigma}\ ;\ c^{+}_{k',\sigma'}\gg\ ]$$

$$+\sum_{\lambda_1q_1}\phi_{\lambda_1,q_1}[\ \delta_{q,-q_1}\,\delta_{\lambda_1,\lambda+q}\ll\!\!\ll a^{+}_{\lambda_1+q_1}\ c_{k+\lambda\downarrow}\ ;\ c^{+}_{k',\sigma'}\gg$$

$$-\ \delta_{\lambda_1,\lambda+q}\ll\!\!\ll a^{+}_{\lambda_1+q_1}\ b_q\ b^{+}_{-q_1}\ c_{k+\lambda\downarrow}\ ;\ c^{+}_{k',\sigma'}\gg$$

$$+\ \delta_{q,-q_1}\ll\!\!\ll a^{+}_{\lambda_1+q_1}\ a^{+}_{\lambda+q}\ a_{\lambda_1}\ c_{k+\lambda\downarrow}\ ;\ c^{+}_{k',\sigma'}\gg$$

$$-\ \delta_{\lambda_1,\lambda+q}\ll\!\!\ll a_{\lambda_1+q_1}\ b_q\ b_{q_1}\ c_{k+\lambda\downarrow}\ ;\ c^{+}_{k',\sigma'}\gg\ ] \qquad (2.23)$$

In order to reduce the higher order Green functions at suitable stage, we must have some criterion to decouple them. There are various decoupling approximations, but the one we are using is

due to Bogolyubov and Tyablikov.[24] However, as Tahirkheli and ter Haar put it, the procedure of decoupling is at present far from being well understood.[25-31]

The above equation can be reduced by the following approximations

$$- \delta_{\lambda_1,\lambda+q} \ll a^+_{\lambda_1+q_1} b_q b^+_{-q_1} c_{k+\lambda\downarrow} ; c^+_{k',\sigma'} \gg$$

$$\simeq (b_q b^+_q) \ll a^+_\lambda c_{k+\lambda\downarrow} ; c^+_{k',\sigma'} \gg$$

$$\simeq (1 + N^p_q) \ll a^+_\lambda c_{k+\lambda\downarrow} ; c^+_{k',\sigma'} \gg$$

Similarly

$$\delta_{q,-q_1} \ll a^+_{\lambda_1+q_1} a^+_{\lambda+q} a_{\lambda_1} c_{k+\lambda\downarrow} ; c^+_{k',\sigma'} \gg$$

$$\simeq (N^m_{\lambda+q}) \ll a^+_\lambda c_{k+\lambda\downarrow} ; c^+_{k',\sigma'} \gg$$

where (N^p_q) and $N^m_\lambda)$ etc. are the phonon and magnon occupation numbers. Further the average values of (b_q), (b^+_q), (a^+_λ) etc. and the products of two creation or two annihilation operators will be taken as zero.

Thus the equation (2.23) reduces to

$$(w - \epsilon_{k+\lambda} + E^m_{\lambda+q} - E^p_q) \ll c_{k+\lambda\downarrow} a^+_{\lambda+q} b_q ; c^+_{k',\sigma'} \gg$$

$$= \delta_{\lambda q} [(N^m_{\lambda+q}) - (N^p_q)] \ll a^+_\lambda c_{k+\lambda\downarrow} ; c^+_{k',\sigma'} \gg \qquad (2.24)$$

The several other Green functions (involved in equations 2.14 - 2.17) will be of similar structure. It will be expedient to reduce them by approximate decoupling. Hence other higher order Green functions can be written as:

$$(w-\epsilon_{k+\lambda}+E^m_{\lambda-q}+E^b_q)\ \langle\langle c_{k+\lambda\downarrow}\, a^+_{\lambda+q}\, b^+_{-q}\ ;\ c^+_{k',\sigma'}\rangle\rangle$$

$$= -\phi_{\lambda q}\ [\,(1+N^p_q)+(N^m_{\lambda+q})\,]\ \langle\langle a^+_\lambda\, c_{k+\lambda\downarrow}\ ;\ c^+_{k',\sigma'}\rangle\rangle \qquad (2.25)$$

$$(w-\epsilon_{k-\lambda}-E^m_{\lambda-q}-E^p_q)\ \langle\langle c_{k-\lambda\uparrow}\, a_{\lambda-q}\, b_q\ ;\ c^+_{k',\sigma'}\rangle\rangle$$

$$= \phi_{\lambda q}\ [\,(1+N^p_q)+(N^m_{\lambda-q})\,]\ \langle\langle c_{k-\lambda\uparrow} a_\lambda\ ;\ c^+_{k',\sigma'}\rangle\rangle \qquad (2.26)$$

$$(w-\epsilon_{k-\lambda}-E^m_{\lambda-q}+E^p_q)\ \langle\langle c_{k-\lambda\uparrow}\, a_{\lambda-q}\, b^+_{-q}\ ;\ c^+_{k',\sigma'}\rangle\rangle$$

$$= \phi_{\lambda q}\ [\,(N^p_q)-(N^m_{\lambda-q})\,]\ \langle\langle c_{k-\lambda\uparrow}\, a_\lambda\ ;\ c^+_{k',\sigma'}\rangle\rangle \qquad (2.27)$$

$$(w-\epsilon_{k+q}+E^m_{\lambda+q}-E^m_\lambda)\ \langle\langle a^+_{\lambda+q}\, a_\lambda\, c_{k+q,\sigma}\ ;\ c^+_{k',\sigma'}\rangle\rangle$$

$$= \phi_{\lambda q}\ [\,(N^m_{\lambda+q})-(N^m_\lambda)\,]\ \langle\langle c_{k+q,\sigma}\, b^+_q\ ;\ c^+_{k',\sigma'}\rangle\rangle \qquad (2.28)$$

$$(w-\epsilon_{k-q}+E^m_{\lambda-q}-E^m_\lambda)\ \langle\langle a^+_{\lambda-q}\, a_\lambda\, c_{k-q,\sigma}\ ;\ c^+_{k',\sigma'}\rangle\rangle$$

$$= \phi_{\lambda q}\ [\,(N^m_{\lambda-q})-(N^m_\lambda)\,]\ \langle\langle c_{k-q,\sigma}\, b_q\ ;\ c^+_{k',\sigma'}\rangle\rangle \qquad (2.29)$$

$$(w-\epsilon_{k+\lambda-q}+E^m_\lambda-E^p_q)\ \langle\langle c_{k+\lambda-q\downarrow}\, a^+_\lambda\ b_q\ ;\ c^+_{k',\sigma'}\rangle\rangle$$

$$= A_q\ [\,(1+N^p_q)-(n_{k+\lambda-q\downarrow})\,]\ \langle\langle c_{k+\lambda\downarrow}\, a^+_\lambda\ ;\ c^+_{k',\sigma'}\rangle\rangle \qquad (2.30)$$

$$(w-\epsilon_{k+\lambda+q} + E^m_\lambda + E^p_q) \ll c_{k+\lambda+q\downarrow}\, a^+_\lambda b^+_q \,;\, c^+_{k',\sigma'} \gg$$

$$= A_q \left[(N^p_q) + (n_{k+\lambda+q\downarrow}) \right] \ll c_{k+\lambda\downarrow}\, a^+_\lambda \,;\, c^+_{k',\sigma'} \gg \qquad (2.31)$$

$$(w-\epsilon_{k-\lambda-q} - E^m_\lambda - E^p_q) \ll c_{k-\lambda-q\uparrow}\, a_\lambda\, b_q \,;\, c^+_{k',\sigma'} \gg$$

$$= A_q \left[(1+N^p_q) - (n_{k-\lambda-q\uparrow}) \right] \ll c_{k-\lambda\uparrow} a_\lambda \,;\, c^+_{k',\sigma'} \gg \qquad (2.32)$$

$$(w-\epsilon_{k-\lambda+q} - E^m_\lambda + E^p_q) \ll c_{k-\lambda+q\uparrow}\, a_\lambda\, b^+_q \,;\, c^+_{k',\sigma'} \gg$$

$$= A_q \left[(N^p_q) + (n_{k-\lambda+q\uparrow}) \right] \ll c_{k-\lambda\uparrow} a_\lambda \,;\, c^+_{k',\sigma'} \gg \qquad (2.33)$$

$$(w-\epsilon_{k+q+\lambda} + E^m_\lambda + E^p_q) \ll c_{k+q+\lambda\downarrow} b^+_q\, a^+_\lambda \,;\, c^+_{k',\sigma'} \gg$$

$$= - J_m(k,\lambda) \left[(N^m_\lambda) + (n_{k+q+\lambda\downarrow}) \right] \ll c_{k+q\uparrow} b^+_q \,;\, c^+_{k',\sigma'} \gg \qquad (2.34)$$

$$(w-\epsilon_{k+q-\lambda} - E^m_\lambda + E^p_q) \ll c_{k+q-\lambda\uparrow} b^+_q a_\lambda,\, c^+_{k',\sigma'} \gg$$

$$= - J_m(k,\lambda) \left[(1+N^m_\lambda) - (n_{k+q-\lambda\uparrow}) \right] \ll c_{k+q\downarrow} b^+_q, ; c^+_{k',\sigma'} \gg \qquad (2.35)$$

$$(w-\epsilon_{k-q+\lambda} + E^m_\lambda - E^p_q) \ll c_{k-q+\lambda\downarrow} b_q a^+_\lambda ; \, c^+_{k',\sigma'} \gg$$

$$= - J_m(k,\lambda) \left[(N^m_\lambda) + (n_{k-q+\lambda\uparrow}) \right] \ll c_{k-q\uparrow} b_q \,;\, c^+_{k',\sigma'} \gg \qquad (2.36)$$

$$(w-\epsilon_{k-q-\lambda} - E^m_\lambda - E^p_q) \ll c_{k-q-\lambda\uparrow} a_\lambda b_q \,;\, c^+_{k',\sigma'} \gg$$

$$= - J_m(k,\lambda) \left[(1+N^m_\lambda) - (n_{k-q-\lambda\uparrow}) \right] \ll c_{k-q\downarrow} b_q ; c^+_{k',\sigma'} \gg \qquad (2.37)$$

EVALUATION OF GREEN FUNCTIONS

A) Electron Green Functions

We substitute the relevant higher order Green functions (cf. equations 2.24 to 2.37) in the appropriate places in the equations (2.14 to 2.17). Then the three operator Green functions are obtained in a compact form after certain decoupling approximations namely (see equation 2.14, for example)

$$\delta_{k_1,k+\lambda} \ll c_{k_1-\lambda_1\sigma} a^+_\lambda a_{\lambda_1} \; ; \; c^+_{k',\sigma'} \gg \; = \; (N^m_\lambda) \ll c_{k\sigma} \; ; \; c^+_{k',\sigma'} \gg$$

$$\delta_{\lambda\lambda_1} \ll c_{k+\lambda\downarrow} c^+_{k_1\downarrow} c_{k_1-\lambda_1\uparrow} \; ; \; c^+_{k',\sigma'} \gg \; = 1+(n_{k+\lambda\downarrow}) \ll c_{k\uparrow} \; ; c^+_{k',\sigma'} \gg$$

where $(n_{k\sigma})$ is the average value of the electrons occupation number in the state $|k\sigma\rangle$. Thus equation (2.14) reduces to

$$[w-\epsilon_{k+\lambda}+E^m_\lambda - \bar{A}(w,k+\lambda\downarrow) - \bar{\phi}(w,k+\lambda)] \ll c_{k+\lambda\downarrow} a^+_\lambda \; ; \; c^+_{k',\sigma'} \gg$$

$$= - J_m(k,\lambda) \; [(N^m_\lambda) + (n_{k+\lambda\downarrow})] \ll c_{k\uparrow} \; ; \; c^+_{k',\sigma'} \gg \qquad (2.38)$$

where

$$\bar{A}(w,k+\lambda\downarrow) = \sum_q A^2_q \left[\frac{(1+N^p_q)-(n_{k+\lambda-q\downarrow})}{w-\epsilon_{k+\lambda-q}+E^m_\lambda - E^p_q} - \frac{(N^p_q)-(n_{k+\lambda+q\downarrow})}{w-\epsilon_{k+\lambda+q}+E^m_\lambda+E^p_q} \right]$$

..... (2.38)'

$$\bar{\phi}(w,k+\lambda) =$$

$$= \sum_q \phi^2_{\lambda q} \left[\frac{(N^m_{\lambda+q}) - (N^p_q)}{w-\epsilon_{k+\lambda}+E^m_{\lambda+q}-E^p_q} - \frac{(1+N^p_q) + (N^m_{\lambda+q})}{w-\epsilon_{k+\lambda}+E^m_{\lambda+q}+E^p_q} \right] \qquad (2.38)''$$

Equation (2.15) reduces to

$$[w-\epsilon_{k-\lambda}-E^m_\lambda-\bar{A}(w,k-\lambda) + \bar{\phi}(w,k-\lambda)] \langle\langle c_{k-\lambda\uparrow} a_\lambda ; c^+_{k',\sigma'} \rangle\rangle$$

$$= - J_m(k,\lambda) [(1-n_{k-\lambda\uparrow}) + (N^m_\lambda)] \langle\langle c_{k\downarrow} ; c^+_{k',\sigma'} \rangle\rangle \qquad (2.39)$$

where

$$\bar{A}(w,k-\lambda\uparrow) =$$

$$= \sum_q A^2_q \left[\frac{(1+N^p_q)-(n_{k-\lambda-q\uparrow})}{w-\epsilon_{k-\lambda-q}-E^m_\lambda-E^p_q} + \frac{(N^p_q) + (n_{k-\lambda+q\uparrow})}{w-\epsilon_{k-\lambda+q}-E^m_\lambda+E^p_q} \right] \qquad (2.39)'$$

and

$$\bar{\phi}(w,k-\lambda) =$$

$$= \sum_q \phi^2_{\lambda q} \left[\frac{(N^m_{\lambda+q})+(1+N^p_q)}{w-\epsilon_{k-\lambda}-E^m_{\lambda-q}-E^p_q} + \frac{(N^p_q) - (N^m_{\lambda-q})}{w-\epsilon_{k-\lambda}-E^m_{\lambda-q}+E^p_q} \right] \qquad (2.39)''$$

Also equation (2.16) will reduce to

$$[w-\epsilon_{k+q}-E^p_q - \bar{J}(w,k+q\downarrow) - \bar{\phi}(w,k+q)] \langle\langle c_{k+q\uparrow} b^+_q ; c^+_{k',\sigma'} \rangle\rangle$$

$$= A_q [(N^p_q) + (n_{k+q\uparrow})] \langle\langle c_{k\uparrow} ; c^+_{k',\sigma'} \rangle\rangle \qquad (2.40)$$

and for $\sigma = \downarrow$

$$[\, w-\epsilon_{k+q} - E^p_q - \bar{J}(w,k+q\uparrow) - \bar{\phi}(w,k+q)]\langle\langle c_{k+q\downarrow} b^+_q \,;\, c^+_{k',\sigma'}\rangle\rangle$$

$$= A_q\,[\,(N^p_q) + (n_{k+q\downarrow})\,]\langle\langle c_{k\downarrow}\,;\, c^+_{k',\sigma'}\rangle\rangle \qquad (2.41)$$

where

$$\bar{J}(w,k\pm q\downarrow) = \sum_\lambda J^2_m(k,\lambda)\ \frac{(N^m_\lambda)+[(n_{k\pm q+\lambda\downarrow})]}{w-\epsilon_{k\,\pm\,q-\lambda} - E^m_\lambda \pm E^p_q} \qquad (2.41)'$$

$$\bar{J}(w,k\pm q\uparrow) = \sum_\lambda J^2_m(k,\lambda)\ \frac{(1+N^m_\lambda) - (n_{k\pm q-\lambda\uparrow})}{w-\epsilon_{k\,\pm\,q-\lambda} - E^m_\lambda \pm E^p_q} \qquad (2.41)''$$

and

$$\bar{\phi}(w,k\pm q) = \sum_\lambda \phi^2_{\lambda q}\ \frac{(N^m_{\lambda+q}) - (N^m_\lambda)}{w-\epsilon_{k\pm q} + E^m_{\lambda\pm q} - E^m_\lambda} \qquad (2.41)'''$$

Finally the equation (2.17) reduces to

$$[\, w-\epsilon_{k-q} - E^p_q - \bar{J}(w,k-q\downarrow) - \bar{\phi}(w,k-q)\,]\langle\langle c_{k-q\uparrow} b_q \,;\, c^+_{k',\sigma'}\rangle\rangle$$

$$= A_q\ [\,(1+N^p_q) - (n_{k-q\uparrow})\,]\langle\langle c_{k\uparrow}\,;\, c^+_{k',\sigma'}\rangle\rangle \qquad (2.42)$$

and

$$[\, w-\epsilon_{k-q} - E^p_q - \bar{J}(w,k-q\uparrow) - \bar{\phi}(w,k-q)\,]\langle\langle c_{k-q\downarrow} b_q \,;\, c^+_{k',\sigma'}\rangle\rangle$$

$$= A_q\ [\,(1+N^p_q) - (n_{k-q\downarrow})\,]\langle\langle c_{k\downarrow}\,;\, c^+_{k',\sigma'}\rangle\rangle \qquad (2.43)$$

Making use of the three operator Green functions given in equations (2.38) to (2.43) we can evaluate the appropriate electron Green functions from the equation (2.11). We write all three operator Geeen functions in terms of single particle Green functions (two operator). We get (if we take the diagonal terms i.e. k = k' and $\sigma = \sigma'$)

$$\ll c_{k\uparrow} ; c^{+}_{k\uparrow} \gg = \frac{1}{2\pi\ (w-\epsilon_k - M_{k\downarrow} - P_{k\uparrow})} \tag{2.44}$$

$$\ll c_{k\downarrow} ; c^{+}_{k\downarrow} \gg = \frac{1}{2\pi\ (w-\epsilon_k - M_{k\uparrow} - P_{k\downarrow})} \tag{2.45}$$

where the symbols $\bar{M}_{k\sigma}$ and $\bar{P}_{k\sigma}$ have the following explicit forms

$$\bar{M}_{k\downarrow} = \frac{\sum_{\lambda} J^2_m(k,\lambda)\ [\ (n_{k+\lambda\downarrow}) + (N^m_\lambda)\ ]}{w-\epsilon_{k+\lambda}+E^m_\lambda - \bar{A}(w,k+\lambda\downarrow) - \bar{\phi}(w,k+\lambda)} \tag{2.46}$$

$$\bar{M}_{k\uparrow} = \frac{\sum_{\lambda} J^2_m(k,\lambda)\ [(N^m_\lambda + 1) - (n_{k-\lambda\uparrow})]}{w-\epsilon_{k-\lambda} - E^m_\lambda - \bar{A}(w,k-\lambda\uparrow) - \bar{\phi}(w,k-\lambda)} \tag{2.47}$$

$$\bar{P}_{k\uparrow} = \sum_{q} A^2_q \left[\frac{[\ (N^p_q) + (n_{k+q\uparrow})\ ]}{w-\epsilon_{k+q}+E^p_q-\bar{J}(w,k+q\downarrow) - \bar{\phi}(w,k+q)} + \right.$$

$$\left. + \frac{[(1 + N^p_q) - (n_{k-q\uparrow})]}{w-\epsilon_{k-q} - E^p_q - \bar{J}(w,k-q\downarrow) - \bar{\phi}(w,k-q)} \right] \tag{2.48}$$

$$\bar{P}_{k\downarrow} = \sum_q A_q^2 \left[\frac{[(N_q^p) + (n_{k+q\downarrow})]}{w-\epsilon_{k+q} + E_q^p - \bar{J}(w,k+q\uparrow) - \bar{\phi}(w,k+q)} + \right.$$

$$\left. + \frac{[(1+N_q^p) - (n_{k-q\downarrow})]}{w-\epsilon_{k-q} - E_q^p - \bar{J}(w,k-q\uparrow) - \bar{\phi}(w,k-q)} \right] \qquad (2.49)$$

Thus we have obtained the electron Green functions (equations 2.44 and 2.45) in terms of $M_{k\sigma}$ and $P_{k\sigma}$ which in turn contain the factors involving $\phi_{\lambda q}$ which are due to phonon-magnon interaction.

We have not indicated the spin index σ associated with the electron energies. The nature of the index will be clear from the spin index associated with electron creation and annihilation operator involved in the Green functions.

B) Evaluation of Phonon Green Function

The relevant three operator Green functions in equation (2.18) and (2.19) are evaluated first. We use following decoupling approximation in equation (2.18) e.g.

$$\langle\langle c^+_{k_1\downarrow} c_{k\uparrow} a_\lambda ; b^+_q \rangle\rangle = (c_{k_1\downarrow}, c_{k\uparrow}) \langle\langle a_\lambda ; b^+_q \rangle\rangle = 0$$

Further

$$\langle\langle c^+_{k\sigma} c_{k\sigma} b_q ; b^+_q \rangle\rangle = (c^+_{k\sigma}, c_{k\sigma}) \langle\langle b_q ; b^+_q \rangle\rangle$$

$$= (n_{k\sigma}) \langle\langle b_q ; b^+_q \rangle\rangle$$

and we neglect Green functions of the type

$$\langle\langle c^{+}_{k_1\sigma}\, c_{k\sigma}\; b^{+}_{q}\; ;\; b^{+}_{q} \rangle\rangle$$

Thus equation (2.18) reduces to

$$(w-\epsilon_k + \epsilon_{k-q}) \langle\langle c^{+}_{k-q,\sigma}\, c_{k\sigma}\; ;\; b^{+}_{q} \rangle\rangle$$

$$= A_q [(n_{k-q,\sigma}) - (n_{k\sigma})] \langle\langle b_q \; ;\; b^{+}_{q} \rangle\rangle \qquad (2.50)$$

Likewise equation (2.19) reduces to

$$(w-E^{m}_{\lambda-q}-E^{m}_{\lambda}) \langle\langle a^{+}_{\lambda-q}a_{\lambda};b^{+}_{q} \rangle\rangle = \phi_{\lambda q}[N^{m}_{\lambda-q})-(N^{m}_{\lambda})] \langle\langle b_q \; ;\; b^{+}_{q} \rangle\rangle$$

.... (2.51)

Substituting equations (2.50) and (2.51) in equation (2.12) we get

$$\langle\langle b_q \; ;\; b^{+}_{q} \rangle\rangle = \frac{1}{2\pi} \; \frac{1}{w-E^{p}_{q} - \bar{A}(w,q) - \bar{\phi}(w,q)} \qquad (2.52)$$

where

$$\bar{A}(w,q) = \sum_{k,\sigma} A^{2}_{q} \; \frac{[(n_{k-q,\sigma}) - (n_{k\sigma})]}{w+ \epsilon_k + \epsilon_{k-q}} \qquad (2.53)$$

and

$$\bar{\phi}(w,q) = \sum_{\lambda} \phi^{2}_{\lambda q} \; \frac{[(N^{m}_{\lambda-q}) - (N^{m}_{\lambda})]}{w + E^{m}_{\lambda-q} - E^{m}_{\lambda}} \qquad (2.54)$$

Thus in the equation (2.52) we have obtained a new expression for the phonon Green functions. The new term $\phi(w,q)$, in the denominator comes due to phonon-magnon interaction.

c. Evaluation of the Magnon Green Function

In evaluating equation (2.20) we neglect those higher order Green functions which on decoupling average to the types

$\langle\langle c^{+}_{k_1\uparrow} c_{k\downarrow}\ b_q\ ;\ a^{+}_{\lambda}\rangle\rangle$, and further we have

$$\langle\langle c^{+}_{k-\lambda\sigma} c_{k-\lambda\sigma} a_{\lambda}\ ;\ a^{+}_{\lambda}\rangle\rangle = (n_{k-\lambda,\sigma})\ \langle\langle a_{\lambda},\ a^{+}_{\lambda}\rangle\rangle$$

Thus we get

$$(w-\epsilon_k+\epsilon_{k-\lambda})\ \langle\langle c_{k-\lambda\uparrow} c_{k\downarrow};\ a^{+}_{\lambda}\rangle\rangle$$

$$= -J_m(k,\lambda)\ [(n_{k-\lambda\uparrow}) - (n_{k\downarrow})]\ \langle\langle a_{\lambda}\ ;\ a^{+}_{\lambda}\rangle\rangle \qquad (2.55)$$

Similarly with an analogous decoupling approximation we reduce equation (2.21) to

$$(w- E^{p}_{q} - E^{m}_{\lambda-q})\ \langle\langle a_{\lambda-q} b_q\ ;\ a^{+}_{\lambda}\rangle\rangle$$

$$= \phi_{\lambda q}\ [(1+N^{p}_{q}) + (N^{m}_{\lambda-q})]\ \langle\langle a_{\lambda}\ ;\ a^{+}_{\lambda}\rangle\rangle \qquad (2.56)$$

and finally equation (2.22) reduces to

$$(w + E^{p}_{q} - E^{m}_{\lambda-q})\ \langle\langle a_{\lambda-q}\ b^{+}_{-q}\ ;\ a^{+}_{\lambda}\rangle\rangle$$

$$= \phi_{\lambda q}\ [(N^{p}_{q}) - (N^{m}_{\lambda-q})]\ \langle\langle a_{\lambda}\ ;\ a^{+}_{\lambda}\rangle\rangle \qquad (2.57)$$

Substituting equations (2.55), (2.56) and (2.57) into the equation (2.13), we get

$$\langle\langle a_\lambda , a_\lambda^+ \rangle\rangle = \frac{1}{2\pi} \frac{1}{w - E_\lambda^m - \bar{J}(w,\lambda) - \bar{\phi}(w,\lambda)} \qquad (2.58)$$

where

$$J(w,\lambda) = \sum_k J_m^2(k,\lambda) \frac{(n_{k-\lambda\uparrow}) - (n_{k\downarrow})}{w - \epsilon_k + \epsilon_{k-\lambda}} \qquad (2.59)$$

and

$$\phi(w,\lambda) = \sum_q \phi_{\lambda q}^2 \left[\frac{(1+N_q^p) + (N_{\lambda-q}^m)}{w - E_{\lambda-q}^m - E_q^p} + \frac{(N_q^p) - (N_{\lambda-q}^m)}{w - E_{\lambda-q}^m + E_q^p} \right]$$

..... (2.60)

Thus, we have obtained electron, phonon and magnon Green functions after incorporating the phonon-magnon interaction.

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C H A P T E R - III

S E C T I O N - I

C H A P T E R - III

SECTION - 1

We have obtained new expressions for electron, phonon and magnon Green functions. Hence we are now able to ascertain the renormalised single particle energies of the respective particles. In what follows, it is shown that the new terms $\emptyset$'s coming in the electron-phonon and magnon Green functions are quite important. In point of fact, phonon-magnon interaction gives rise to some very interesting properties.

First, we consider properties of the electrons dressed due to the simultaneous presence of phonons and magnons. In this section, we derive some physical properties of electrons following from the Green functions (2.44) and (2.45). We shall be mainly concerned with the energy renormalisation, spin polarisation, effective mass and life time. For this purpose it is convenient to rewrite the Green functions (i.e. 2.44 and 2.45) in the following form

$$\frac{1}{2\pi}\left[\ll c_{k\uparrow}\,;\, c^{+}_{k\uparrow}\gg\right]^{-1} = w - \epsilon_k - \sum(w, k\uparrow) \qquad (3.1)$$

and

$$\frac{1}{2\pi}\left[\ll c_{k\downarrow}\,,\, c^{+}_{k\downarrow}\gg\right]^{-1} = w - \epsilon_k - \sum(w, k\downarrow) \qquad (3.2)$$

where $\sum(w,k\uparrow)$ and $\sum(w,k\downarrow)$ are the self energies of the conduction electrons with spin up and down respectively. More explicitly

$$\sum(w,k\uparrow) = \bar{M}_{k\downarrow} + \bar{P}_{k\uparrow} \tag{3.3}$$

$$\sum(w,k\downarrow) = \bar{M}_{k\uparrow} + \bar{P}_{k\downarrow} \tag{3.4}$$

The self energy and in turn $\bar{M}_{k\sigma}$ and $\bar{P}_{k\sigma}$ are complex quantities and it is expedient for further calculations to separate the real and imaginary parts by rationalisation.

Following Zubarev[1] we define the spectral density function by

$$J_k(w) = i\left[\frac{\langle\langle c_{k\sigma}, c^{+}_{k\sigma}\rangle\rangle_{w+i\epsilon} - \langle\langle c_{k\sigma}, c^{+}_{k\sigma}\rangle\rangle_{w-i\epsilon}}{(e^{\beta w}+1)}\right] \tag{3.5}$$

Now we have

$$M_{k,\sigma}(w \pm i\epsilon) = \bar{M}_{k\sigma}(w) \mp i\,\overline{m_k}(w) \tag{3.6}$$

and

$$P_k(w \pm i\epsilon) = \bar{P}_{k\sigma}(w) \mp i\,\overline{P_{k\sigma}}(w) \tag{3.7}$$

where $\bar{M}_{k\sigma}$ and $\bar{P}_{k\sigma}$, $\overline{m_k}(w)$ and $\overline{P_k}(w)$ are the real and

imaginary parts of the parameters noted in equations (3.3) and (3.4) (cf equations 2.46 - 2.49). The explicit forms of the two spin states are

$$J_{k\downarrow}(w) = \frac{1}{\pi} \quad \frac{(\lceil m_{k\downarrow} + \lceil P_{k\uparrow}) (e^{\beta w} + 1)^{-1}}{(w-\epsilon_k - \bar{M}_{k\downarrow} - \bar{P}_{k\uparrow})^2 + (\lceil m_{k\downarrow} + \lceil P_{k\uparrow})^2} \tag{3.8}$$

$$J_{k\downarrow}(w) = \frac{1}{\pi} \quad \frac{(\lceil m_{k\uparrow} + \lceil P_{k\downarrow}) (e^{\beta w} + 1)^{-1}}{(w-\epsilon_k - \bar{M}_{k\uparrow} - \bar{P}_{k\downarrow})^2 + (\lceil m_{k\uparrow} + \lceil P_{k\downarrow})^2} \tag{3.9}$$

In all the above equations we have taken $\epsilon \rightarrow 0$

In order to simplify the above expressions we utilise following compact notations

$$M_k(w+i\epsilon) = \frac{\sum_{\lambda} J_m^2(k,\lambda) [(n_{k+\lambda}) + (N_\lambda^m)]}{A + iB} \tag{3.10}$$

After rationalising we get real and imaginary parts i.e. $\bar{M}_{k\downarrow}(w)$ and $\lceil m,k\downarrow$

$$\bar{M}_{k\downarrow}(w) = \frac{\sum_{\lambda} J_m^2 (k,\lambda) [(n_{k+\lambda\downarrow}) + (N_\lambda^m)] A}{A^2 + B^2} \tag{3.11}$$

and

$$\overline{\Gamma}_{m_{k\downarrow}}(w) = \frac{\sum_{\lambda} J_m^2(k,\lambda)\ [\ (n_{k+\lambda\downarrow}) + (N_\lambda^m)\ ]\ B}{A^2 + B^2} \qquad (3.12)$$

where

$$A = w - \epsilon_{k+\lambda} + E_\lambda^m - \overline{A}(w, k+\lambda\downarrow) - \overline{\emptyset}(w, k+\lambda) \qquad (3.13)$$

and

$$B = A_i(w,\ k+\lambda\downarrow) + \emptyset_i(w,\ k+\lambda) \qquad (3.14)$$

where bar and suffix i represent the principal and imaginary parts of the respective expressions. (We have taken $\epsilon \to 0$ in equation (2.14) [cf. equations (2.38)' and (2.38)" for details].

A_i and $\emptyset_i$ are obtained with the help of equations (3.8) and (3.9) respectively. Similarly for $\overline{M}_{k\uparrow}$ and $\overline{\Gamma}_{m_{k\uparrow}}$ we get

$$A = w - \epsilon_{k-\lambda} - E_\lambda^m - \overline{A}(w, k-\lambda\uparrow) - \overline{\emptyset}(w, k-\lambda) \qquad (3.15)$$

$$B = A_i(w,\ k-\lambda\uparrow) - \emptyset(w, k-\lambda) \qquad (3.16)$$

Similarly

$$P_{k\uparrow}(w + i\epsilon) =$$

$$= \sum_q A_q^2 \left[\frac{[\ (N_q^p) + (n_{k+q\uparrow})\ ]}{A' + iB'} + \frac{[\ (1+N_q^p) - (n_{k-q\uparrow})\ ]}{A'' + iB''} \right]$$

.....(3.17)

$$A' = w-\epsilon_{k+q} + E_q^p - \bar{J}(w,k+q\downarrow\ ) - \bar{\phi}(w,k+q) \tag{3.18}$$

$$B' = J_i(w,k+q\downarrow\ ) + \phi_i(w,k+q) \tag{3.19}$$

$$A'' = w-\epsilon_{k-q} - E_q^p - \bar{J}(w,k-q\downarrow\ ) - \bar{\phi}(w,k-q) \tag{3.20}$$

$$B'' = J_i(w,k-q\downarrow\ ) + \phi_i(w,k-q\ ) \tag{3.21}$$

Similarly we get A', B' and A", B" for $P_{k\downarrow}(w+i\epsilon\ )$ as follows.

$$A' = w-\epsilon_{k+q} + E_q^p - \bar{J}(w,k+q\uparrow) - \bar{\phi}(w,k+q) \tag{3.22}$$

$$B' = J_i(w,k+q\uparrow\ ) + \phi_i(w,k+q) \tag{3.23}$$

$$A'' = w-\epsilon_{k-q} + E_q^p - \bar{J}(w,k-q\uparrow\ ) - \bar{\phi}(w,k-q) \tag{3.24}$$

$$B'' = J_i(w,k-q\uparrow\ ) + \phi_i(w,k-q\ ) \tag{3.25}$$

Substituting the relevant quantities noted above in equations (3.8) and (3.9), we can evaluate the spectral density. With all these substitutions the expressions become extremely complicated. Therefore, to extract some meaningful results from them, we employ some approximations in order to simplify them. For example, for determining the occupation number we ignore damping as is done generally. The occupation number is determined with the help of these spectral density functions.

$$(n_{k\sigma}) = \frac{1}{\pi} \int_{\infty}^{\infty} J_{k\sigma}(w)\, dw \qquad (3.26)$$

Using this definition and neglecting damping in equations (3.8) and (3.9) we get

$$(n_{k\uparrow}) = \frac{1}{e^{\beta(\widetilde{w}_{\uparrow} - E_f)} + 1} \qquad (3.27)$$

and

$$(n_{k\downarrow}) = \frac{1}{e^{\beta(\widetilde{w}_{\downarrow} - E_f)} + 1} \qquad (3.28)$$

We have used the following identity[2] in arriving the above result

$$\lim \epsilon \to 0 \quad \frac{\epsilon}{x^2 + \epsilon^2} = \pi\, \delta(x)$$

and $\widetilde{w}_{\uparrow}$ and $\widetilde{w}_{\downarrow}$ are the renormalised single particle electron energies for the up and down spin states

$$\widetilde{w}_{\uparrow} = \epsilon_k + \overline{M}_{k\downarrow} + \overline{P}_{k\uparrow} \qquad (3.29)$$

$$\widetilde{w}_{\downarrow} = \epsilon_k + \overline{M}_{k\uparrow} + \overline{P}_{k\downarrow} \qquad (3.30)$$

We rewrite the expressions (2.46) and (2.49) for $\overline{M}_{k\sigma}$ and $\overline{P}_{k\sigma}$ with the principal values wherever necessary, which shall give correct expressions for $\overline{M}_{k\sigma}$ and $\overline{P}_{k\sigma}$ involved in equations (3.29) and (3.30).

$$\bar{M}_{k\downarrow} = \sum_{\lambda} \frac{J_m^2(k,\lambda)\ [(n_{k+\lambda\downarrow}) + (N_\lambda^m)\ ]}{w-\epsilon_{k+\lambda} - E_\lambda^m - \bar{A}(w,k+\lambda\downarrow) - \bar{\phi}(w,k+\lambda)} \tag{3.31}$$

$$\bar{M}_{k\uparrow} = \sum_{\lambda} \frac{J_m^2(k,\lambda)\ [(1+N_\lambda^m) - (n_{k-\lambda\uparrow})\ ]}{w-\epsilon_{k-\lambda} - E_\lambda^m - \bar{A}(w,k-\lambda\uparrow) - \bar{\phi}(w,k-\lambda)} \tag{3.32}$$

$$\bar{P}_{k\uparrow} = \sum_{q} A_q^2 \left[\frac{[\ (N_q^p) - (n_{k+q\uparrow})\ ]}{w-\epsilon_{k+q}+E_q^p-\bar{J}(w,k+q\downarrow) - \bar{\phi}(w,k+q)} + \right.$$

$$\left. + \frac{[\ (N_q^p + 1) - (n_{k-q\uparrow})\ ]}{w-\epsilon_{k-q} - E_q^p-\bar{J}(w,k-q\downarrow) - \bar{\phi}(w,k-q)} \right] \tag{3.33}$$

$$\bar{P}_{k\downarrow} = \sum_{q} A_q^2 \left[\frac{[\ (N_q^p) + (n_{k-q\downarrow})\ ]}{w-\epsilon_{k-q} + E_q^p - \bar{J}(w,k+q\uparrow) - \bar{\phi}(w,k+q)} + \right.$$

$$\left. + \frac{[\ (1+N_q^p) - (n_{k-q\downarrow})\ ]}{w-\epsilon_{k-q} - E_q^p - \bar{J}(w,k-q\uparrow) - \bar{\phi}(w,k-q)} \right] \tag{3.34}$$

To evaluate equations (3.31) to (3.34) we transform summations to integration as follows[3]

$$\sum_{k} \rightarrow \frac{\Omega}{(2\pi)^3} \int d^3k \qquad (3.35)$$

where the direct volume Ω (v) contains N primitive cells. In order to carry out the integrations, we shall require the explicit forms of the various coupling coefficients. As is customary we assume that[4-5]

$$J_m(k,\lambda) = \left(\frac{2S}{N}\right)^{1/2} J_o \qquad (3.36)$$

does not depend upon the electron and magnon wave vectors (which is not the case really[6-7]). For the electron-phonon interaction involving acoustic phonons, we have[8-11]

$$A_q = \frac{C_1}{\sqrt{N}} \left(\frac{\hbar}{2v_s M}\right) q^{1/2} \qquad (3.37)$$

where C_1 is a constant having dimensions of energy and is equal[8] to 1 to 10 eV. v_s is the velocity of sound M is the atomic mass.

$\phi_{\lambda q}$ for phonon-magnon coupling has been derived earlier.[12-14]. Its form in the long-wave length approximation is given by

$$\phi_{\lambda q} = \frac{8}{\sqrt{N}} \left(\frac{\hbar}{2w_q M}\right)^{1/2} 2\lambda q a^2 \cos\theta_{\lambda q} \left[2S\, e_{J(R_h)}\right] \qquad (3.38)$$

where S is the spin of magnetic atom and $e_{J(R_h)}$ is a

parameter involving modulated exchange integral due to crystal field oscilations. It has the dimensions of force. Details are given in the earlier references. Further the unperturbed electron, magnon and phonon (acoustic) energies are given by (reinstating h)

$$\left.\begin{aligned} \epsilon_k &= \hbar^2 k^2 / 2m \\ E^m &= I\lambda^2 + \hbar w_o \\ E^p_q &= \hbar w_q = \hbar v_s q \end{aligned}\right\} \qquad (3.39)$$

In the absence of magnetic field $\hbar w_o = 0$ and $E^m_\lambda = I\lambda^2$.

With these forms of coupling coefficients (cf. equations 3.36 to 3.38) and the dispersion relations (3-39), the results after integrations of various quantities involved in equations (3.31) to (3.34) are given by (They are explicitly calculated in the appendices).

$$\bar{A}(w, k \pm\lambda, \sigma) = \frac{c_1^2 m q_m^2 \left(w - \epsilon_{k \pm\lambda, \sigma} \pm E^m_\lambda \right)}{16\pi^2 n \hbar \epsilon_k v_s M} \qquad (3.40)$$

where σ is the spin index and q_m is the maximum Debye wave-vector for the phonon and is given by π/a, n is the density of electrons per unit volume.

$$J(w,k\pm q\uparrow) = -\frac{J_o^2\, Sa^3 m}{4\pi^{3/2}\hbar^2 \epsilon_k \sqrt{\beta I}} \quad (w-\epsilon_{k\pm q} \pm E_q^p) \qquad (3.41)$$

and

$$\bar{J}(w,\ k \pm q\downarrow) =$$

$$= \left[-\frac{J_o^2\, Sa^3 m}{4\pi^{3/2}\hbar^2 \epsilon_k \sqrt{\beta I}} - \frac{5 J_o^2 a^3 m \lambda_m}{4\pi^2 \hbar^2 \epsilon_k} \right] (w-\epsilon_{k\pm q} \pm E_q^p)$$

.... (3.42)

and finally

$$\bar{\phi}(w,k\pm\lambda) = \frac{8a^7 q_m^2 \hbar (2S)^2 \left({}^{e}J(R_h)^2 \right.}{\pi^2 I^2 v_s M} (w-\epsilon_{k\pm\lambda} \pm E_\lambda^m) \qquad (3.43)$$

$$\bar{\phi}(w,k\pm q) = \frac{32a^7 \left[{}^{e}J(R_h)^2 \right] . S^2 \, I\, m \left[(1/2) \right.}{\pi^2 v_s M h \epsilon_k (\beta I)^{5/2}} (w-\epsilon_{k+q} \pm E_q^p)$$

..... (3.44)

Next, we substitute these quantities (cf. equations 3.40 - 3.44) at the appropriate places in the expressions for $\bar{M}_{k\sigma}$ and $\bar{P}_{k\sigma}$ and carry out the relevant summations (integration). It is extremely difficult to perform these integrations under general conditions. However, for some specific situations they can be solved. First we replace 'w' occurring in $\bar{M}_{k\sigma}$ and $\bar{P}_{k\sigma}$ by the unperturbed electron energy i.e. ϵ_k. Further we assume that

$$w - \epsilon_{k+\lambda} + E^{m}_{\lambda} < 2Akq \qquad (3.45)$$

where $A = \hbar^2/2m$. This implies that we are in long wave length (i.e. short magnon wave-vector) $\lambda < k$ region as far as magnons are concerned. (The assumption has been clarified in the appendix).

$$\bar{M}_{k\downarrow} = \frac{- SJ_o^2 a^3 \Gamma(1/2)}{16\pi^2 \epsilon_k (\beta I)^{3/2} \left[1 - \frac{C_1^2 m q_m^2}{16n\pi^2 v_s M h \epsilon_k} - \frac{8a^7 \hbar S^2 e_{J(R_h)}^2 q_m^2}{\pi^2 v_s M I^2}\right]} \qquad \ldots\ldots (3.46)$$

and

$$\bar{M}_{k\uparrow} = \frac{- \left[\frac{SJ_o^2 a^3 \Gamma(1/2)}{16\pi^2 \epsilon_k (\beta I)^{3/2}} + \frac{SJ_o^2 a^3 \lambda_m^3}{12\pi^2 \epsilon_k}\right]}{\left[1 - \frac{C_1^2 m q_m^2}{16n\pi^2 v_s M h \epsilon_k} - \frac{8a^7 S^2 e_{J(R_h)}^2 q_m^2}{\pi^2 v_s M I^2}\right]} \qquad \ldots\ldots (3.47)$$

In the above equations q_m and λ_m are the maximum values for phonon and magnon wave vectors respectively.

Also

$$\bar{P}_{k\uparrow} = \frac{C_1^2 \hbar q_m^4}{16\pi^2 n \epsilon_k v_s M \left[1 - \frac{J_o^2 S a^3 M}{4\pi^{3/2} \hbar^2 \epsilon_k \sqrt{\beta I}}\right]} \qquad (3.48)$$

and

$$\bar{P}_{k\downarrow} = - \frac{C_1^2 \hbar q_m^4}{16\pi^2 n \epsilon_k v_s^M \left[1 - \frac{SJ_o^2 a^3 m}{4\pi^{3/2}\hbar^2 \epsilon_{kv}\sqrt{\beta I}} - \frac{J_o^2 S a^3 m \lambda_m}{4\pi^2 \hbar^2 \epsilon_k} \right]} \qquad (3.49)$$

While calculating $\bar{P}_{k\sigma}$ we have neglected $\bar{\phi}(w, k \pm q)$ as it is extremely negligible. (See appendix).

Estimates and discussion.

In a magnetic crystal when we consider only electron-magnon interaction, the electron Green function is given by[15]

$$G^{\sigma}_{kk} = (2\pi)^{-1} (w - \epsilon_k - \bar{M}_{k\sigma})^{-1} ,$$

and hence the renormalised electron energy is given by

$$w = \epsilon_k + \bar{M}_{k\sigma}$$

where $\bar{M}_{k\sigma}$ is the self energy of the electron in magnetic field. Similarly in phonon field the renormalised electron energy is given by[15]

$$w = \epsilon_k + \bar{P}_{k\sigma} ,$$

In both cases, the energy of electron is reduced and we observe enhancement in effective mass.

In magnetic crystals, in particular ferromagnetic semiconductors, it is imperative to consider simultaneous presence of phonon and magnon field. Hence after setting up the total Hamiltonian (Equation 2.1), we obtained the electron Green functions (cf. equations 2.44 and 2.45) as

$$G_{kk} = \frac{1}{2\pi\,(w - \epsilon_k - \bar{M}_{k\sigma} - \bar{P}_{k\sigma'})}$$

where $\sigma = \sigma' = \pm 1/2$ (i.e. up and down). Hence the renormalised energy of electrons is given by

$$\tilde{w} = \epsilon_k + \bar{M}_{k\sigma} + \bar{P}_{k\sigma'}$$

(cf equations 3.29 and 3.30)

A glance at the explicit expressions for $\bar{M}_{k\sigma}$ and $\bar{P}_{k\sigma}$ (equations 3.46 - 3.49) will show that the correction due to phonon-magnon interaction is appearing in the square bracket denominator. If we calculate these quantities it is clear that they modify the values of mass operators $\bar{M}_{k\sigma}$ and $\bar{P}_{k\sigma}$ appreciably.

For the sake of clarity we write the equation (3.46) in the following form

$$\bar{M}_{k\downarrow} = \frac{-C_m}{\left[1 - \frac{C_1^2 m q_m^2}{16n\pi^2 v_s M h \epsilon_k} + \frac{8a^7 \hbar s^2 \left[{}^{e}J(R_h)\right]^2 q_m^2}{\pi^2 v_s MI^2}\right]} \tag{3.50}$$

where C_m is the electron energy modification due to the electron-magnon (s-d) interaction alone. The second and third terms in the square bracket are the corrections to C_m due to electron-phonon and phonon-magnon interactions. It will be in order to give a rough estimate of the terms involved in the square bracket. We choose the following values (collected from various standard works) for the quantities involved in the equation.

C_1	$= 5 \times 10^{-11}$ ergs	a	$= 5 \times 10^{-8}$ cm
m	$= 10^{-27}$ gms	ϵ_k	$= 5$ eV
v_s	$= 5 \times 10^{-5}$ cm/sec.	${}^{e}J(R_h)$	$= 10^{-6}$ dynes
q_m	$= \pi/a$	I	$= 10^{-29}$ ergs. cm^2
M	$= 5 \times 10^{-23}$ gms	n	$= 10^{22}$ per cm^3

With this we get

$$\frac{C_1^2 m q_m^2}{16n\pi^2 v_s M h \epsilon_k} \simeq 1/3 \tag{3.51}$$

and

$$\frac{8a^7 \hbar s^2 \left[{}^{e}J(R_h)\right]^2 q_m^2}{\pi^2 v_s MI^2} \simeq 1/5 \tag{3.52}$$

1. This shows that the correction due to electron-phonon and phonon-magnon interactions are of comparable magnitude and are important in calculating $\bar{M}_{k\sigma}$, in turn the electron energy as well. This justifies the inclusion of phonon-magnon interaction in the total Hamiltonian of the magnetic crystal.

2. The electron-phonon interaction correction (Equation 3.51) increases the magnitude of $M_{k\sigma}(C_m)$ in equation(3.50) while phonon-magnon interaction (Equation 3.52) reduces it. Thus the energy of electron is further reduced by the electron-phonon interaction and hence the enhancement of effective mass is further supplemented as is mentioned earlier and observed experimentally[19] while the effect of phonon-magnon interaction is to increase the energy of the electron and nullify the enhancement of effective mass (supplemented by the electron-phonon interaction correction). Thus contrary to electron-phonon and electron-magnon interaction the phonon-magnon interaction tends to reduce the effective mass of the electron. But the total effect of taking the simultaneous presence of phonons and magnons is to enhance the effective mass of electron (as generally the quantity in equation (3.51) is greater than that in (3.52). Actually the electron-phonon interaction supplementing the enhancement of effective mass due to electron-magnon interaction has been reported.[18]

3. The modification of $\bar{M}_{k\sigma}$ for $\sigma = \uparrow$ and $\sigma = \downarrow$ is same due to phonon-magnon interaction (cf. equations 6.48 and 6.49)

4. Since we have taken into consideration all the three interactions even $\bar{P}_{k\sigma}$ is different for up and down electron. Phonon-magnon correction to $\bar{P}_{k\sigma}$ is extremely negligible (refer equations 3.48 and 3.49).

5. From equation (3.29) and (3.30) we know that

$$(\tilde{w}_{\uparrow} - \tilde{w}_{\downarrow}) = (\bar{M}_{k\downarrow} - \bar{M}_{k\uparrow}) + (\bar{P}_{k\uparrow} - \bar{P}_{k\downarrow}) \qquad (3.53)$$

which is equal to

$$\frac{2S J_o^2 a^3 m \lambda_m}{\pi^2 \hbar^2 \left[1 - \frac{G_1^2 m q_m^2}{16 n \pi^2 v_s M h \epsilon_k} + \frac{8 a^7 \hbar S^2 ({}^{e}J(R_h))^2 q_m^2}{\pi^2 v_s M I^2} \right]} \qquad (3.53')$$

Thus the energy difference between up and down electrons is decreased due to the third term (due to phonon-magnon interaction) but increased due to electron-phonon correction). On the whole the difference is increased (Polarisation increases).

6. It is clear from equations (3.29) and (3.30) and (3.46 to 3.49) that, there is shift in the electron energy. The effect of electron-phonon and electron-magnon interactions is to reduce the energy of electron and hence to increase the

effective mass. Magnon-phonon interaction has a different effect i.e. to increase the energy and reduce the effective mass. The effective mass is thus enhanced.

We give a rough estimate of energy shift below (estimated from equations (3.46 - 3.49)

$$\bar{M}_{k\downarrow} = \frac{s\,10^{-26}\,T^{3/2}}{\epsilon_k} \qquad (3.54a)$$

$$\bar{M}_{k\uparrow} = \frac{-\,s\,10^{-25}\,T^{3/2}}{\epsilon_k} - \frac{10^{-27}}{\epsilon_k} \qquad (3.54b)$$

$$P_{k\sigma} = -\frac{10^{-24}}{\epsilon_k} \qquad (3.55)$$

In as much as the expressions for $\bar{M}_{k\sigma}$ and $\bar{P}_{k\sigma}$ (3.54 - 3.55) above contain ϵ_k in the denominator, the renormalisation effects are more pronounced for low energy electrons than for high energy electrons. This is to be expected, as more energetic electrons will not be affected by such interactions terms. Also the temperature dependence reflects the consistency of our equations. At low temperature $\bar{M}_{k\sigma}$ will be sensitive to temperatures. At very low temperature the $\bar{P}_{k\sigma}$ terms (arising out of electron-phonon) terms will not depend on temperature. In the low temperature region (say upto and $10^{\circ}K$), the $\bar{M}_{k\sigma}$ term is comparable with $\bar{P}_{k\sigma}$. Hence the mass enhancement due to electron-magnon term may become comparable with that due to electron-phonon interaction. This has been actually observed.[17]

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C H A P T E R - III

S E C T I O N - II

Section - 2

Transport properties

Once we know the relaxation time T_r we can calculate various quantities of interest in the transport theory, such as mobility, conductivity etc. The imaginary parts of the Green functions will help us in calculating Γ_{m_k}, Γ_{p_k} in the equations (3.6 to 3.9) represent the damping terms. It turns out that the relaxation time is related to it by a single relation,(1)

$$\frac{1}{T_r} = \left(\frac{2}{\hbar}\right)\Gamma \qquad (3.56)$$

where $\Gamma = \Gamma_{m_k} + \Gamma_{p_k}$.

With the phonon-magnon interaction part the expression for Γ becomes extremely complicated. If we neglect H_{pm} in the equation (2.1) the calculations for Γ become managable. Also, we are justified in neglecting H_{pm} in the transport phenomena as its effect will be less on the scattering of electrons.

In point of fact, here we have developed Green functions method to obtain the relaxation time T_{sd} (T_{em}), T_{ep} etc. and in turn mobility. When H_{pm} is not there in equation (2.1), we need not solve the Green functions for higher order and hence we can neglect terms such as $\bar{A}(w, k \pm \lambda)$, $\bar{J}(w, k \pm q)$ etc.

[Refer equation (2.38) to (2.41)]. Taking this into account, we get (cf. equation 2.46).

$$M_{k\sigma}(w+i\epsilon) = \sum_{\lambda} \frac{J_m^2(k,\lambda)\,[(n_{k+\lambda\sigma}) + (N_\lambda^m)]}{w - \epsilon_{k+\lambda} + E_\lambda^m + i\epsilon} \qquad (3.57)$$

Hence using the relation[2]

$$\frac{1}{w + i\,\Gamma} = P/w - i\,\pi\,\delta(w)$$

where P denotes the operation of taking principal value of an integral.

We get

$$\Gamma_{m_{k\downarrow}} = \sum_{\lambda} J_m^2(k,\lambda)\,[(n_{k+\lambda\downarrow}) + (N_\lambda^m)]\,\delta(w-\epsilon_{k+\lambda} + E_\lambda^m)$$

Similarly by using equation (2.48) we get

$$\Gamma_{p_{k\uparrow}} = \sum_{q} A_q^2 \{[(N_q^p) + (n_{k+q\uparrow})]\,\delta(w-\epsilon_{k+q} + E_q^p) + [(1+N_q^p) - (n_{k-q\uparrow})]\,\delta(w-\epsilon_{k-q} - E_q^p)\}$$

Hence

$$\Gamma_{\uparrow} = \sum_{\lambda} J_m^2(k,\lambda)\,[(n_{k+\lambda\downarrow}) + (N_\lambda^m)]\,\delta(w-\epsilon_{k+\lambda} + E_\lambda^m) + \sum_{q} A_q^2 \{[(N_q^p) + (n_{k+q\uparrow})]\,\delta(w-\epsilon_{k+q} + E_q^p) + [(1+N_q^p) - (n_{k-q\uparrow})]\,\delta(w-\epsilon_{k-q} - E_q^p)\} \qquad (3.58)$$

and

$$\Gamma_{\downarrow} = \sum_{\lambda} J_m^2(k,\lambda)\,[(n_{k-\lambda\uparrow}+1) + (N_\lambda^m)\,]\,\delta\,(w-\epsilon_{k-\lambda} - E_\lambda^m)$$

$$+ \sum_{q} A_q^2 \{[\,(N_q^p) + (n_{k+q\downarrow})\,]\,\delta\,(w-\epsilon_{k+q} + E_q^p)$$

$$+ [\,(1+N_q^p) - (n_{k-q\downarrow})]\,\delta\,(w-\epsilon_{k-q} - E_q^p)\} \qquad (3.59)$$

We will obtain expressions for relaxation time from the above two equations in the high and low temperatu-re limits.

In the high temperature limit

$$(N_\lambda^m) \simeq \frac{k_\beta T}{I\lambda^2}, \qquad (N_q^p) \simeq \frac{k_\beta T}{\hbar\, v_s q} \qquad (3.60)$$

$$(N_\lambda^m + 1) \simeq N_\lambda^m, \qquad (N_q^p + 1) \simeq (N_q^p) \qquad (3.60')$$

In the low temperature limit

$$\left.\begin{aligned} (N_\lambda^m) &\simeq e^{-\beta I\lambda^2} \\ (N_q^p) &\simeq e^{-\beta(\hbar v_s q)} \end{aligned}\right\} \qquad (3.61)$$

i.e. Boltzmann approximation.

The relaxation time of electrons with spin up $T_{r\uparrow}$ and spin down $T_{r\downarrow}$ turns out to be same in high temperature region. From equation (3.58) after converting the summations into integrations, we get (The integrals of similar type are given in Ehrenrich, Ref. 7).

$$T_r = \frac{\hbar^3}{4ma^3} \frac{1}{\frac{J_o^2 \hbar \sqrt{k_\beta T}}{4I\sqrt{2m}} \log \frac{k_\beta T}{S\mu J} + \frac{C_1^2 \sqrt{2m}\,(k_\beta T)^{3/2}}{2\hbar v_s^2 M}} \qquad (3.62)$$

The two terms in the denominator are due to electron-magnon and electron-phonon interaction respectively. In absence of H_{em} we know that the relaxation time is given by

$$T_r = \frac{\hbar^4 v_s M (k_\beta T)^{-3/2}}{2C_1^2 m^{3/2} a^3} \qquad (3.62')$$

which is a well known temperature dependence law in case of semicondu ctors. Exactly same expression has been obtained by Krishnamurthy-Sinha[3] and some other authors[4-7] by different methods. Hence the mobility due to lattice (acoustic) scattering is proportional to $T^{-3/2}$

The relaxation time due to magnon scattering is proportional to

$$\frac{1}{T^{1/2} \log CT}$$

where C is a constant.

If we substitute and estimate T_r (by selecting various values previously selected) we get

$$T_r = 10^{-32} \frac{1}{10^{-23} T^{1/2} \log 10^{19} T + 5 \times 10^{-20} T^{3/2}} \qquad (3.63)$$

Thus the relaxation time due to magnon scattering is of the order 10^{-8} to 10^{-10}, which is actually the case[8-13]. Hence

$$T_{em} \simeq 10^{-8} \text{ to } 10^{-10}$$

and that due to lattice scattering[14]

$$T_{ep} \simeq 10^{-12}$$

Thus at high temperatures lattice scattering is more dominant than the magnon scattering. Hence, the conductivity is almost due to lattice scattering above $10^{o}K$.

In the low temperature region, we get $T_{r\uparrow} \neq T_{r\downarrow}$. Using the equation (3.59) we get

$$T_{r\uparrow} = \frac{\hbar^3}{4ma^3} \frac{1}{\frac{J_o^2(S)\hbar\sqrt{k_\beta T}}{8\pi^2\sqrt{2m}\ I} + \frac{C_1^2\, m\, k_\beta T}{\hbar v_s M}} \qquad (3.64)$$

$$T_{r\downarrow} = \frac{\hbar^3}{4ma^3} \frac{1}{\frac{J_o^2\sqrt{2mk_\beta}\ T^{1/2}}{4\hbar} - \frac{2(S/\mu J)^2\sqrt{2m}\ J_o^2}{\hbar(k_\beta T)^{3/2}} + \frac{C_1^2\, mk_\beta T}{\hbar v_s M}} \qquad (3.65)$$

Hence the relaxation time due to magnon scattering is proportional to $T^{-1/2}$ and due to lattice scattering is proportional to T^{-1}. Resistivity and conductivity due to s-d

scattering in case of metals have been studied in great detail[15-20], but their temperature dependence at very low temperatures in case of ferromagnetic semiconductors is not known completely. Recently, this has been studied by some authors[20-22]. But the exact temperature dependence at very low temperatures is not known. It depends upon various factors such as types of carriers, impurity injection etc. From equations (3.64) and (3.65) we obtain T^{-1} dependence of conductivity due to lattice scattering alone. Such type of temperature dependence of conductivity is reported in $FeCr_2Se_4$ and $CuCr_2Se_4$ which are ferromagnetic semiconductors[23]. In general the $T^{-3/2}$ dependence of conductivity holds good for a considerable temperature range (above $10^{o}K$).

The approximate values of relaxation time are given by (after substituting relevant values in equation (3.64) and (3.65)).

$$T_{r\uparrow} = \frac{10^{-32}}{6 \times 10^{-20}\, T^{1/2} + 10^{-20}\, T} \qquad (3.66)$$

and

$$T_{r\downarrow} = \frac{10^{-32}}{10^{-21}\, T + 10^{-20}\, T} \qquad (3.67)$$

The order of magnitude of relaxation time is close to experimental values.

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C H A P T E R - III

S E C T I O N - III

Section - 3

Renormalisation of phonon and magnon energies.

Here we obtain expressions for the modified energies of phonons and magnons by the phonon-magnon interaction.

PHONONS

We have obtained expression for phonon Green function given by

$$\ll b_q \; ; \; b_q^+ \gg = \frac{1}{2\pi} \quad \frac{1}{w - E_q^p - \overline{A}(w,q) - \overline{\phi}(w,q)} \qquad (2.52)$$

After neglecting damping, the distribution function (modified) for phonons is given by

$$(N_q^p) = \frac{1}{e^{\beta \tilde{E}_q^p} - 1} \qquad (3.68)$$

where the modified energy of phonon $\tilde{w} = \tilde{E}_q^p$ is given by

$$\tilde{w} = \tilde{E}_q^p = E_q^p + \overline{A}(w, q) + \overline{\phi}(w,q) \qquad (3.69)$$

Explicit forms of $\overline{A}(w,q)$ and $\overline{\phi}(w,q)$ are given in equations (2.53) and (2.54). The energy renormalisation of phonons due to electron-phonon interaction (H_{ep}) i.e. $\overline{A}(w,q)$ is already obtained by several authors[1-6]. The new term $\overline{\phi}(w,q)$ is the correction due to phonon-magnon interaction. After evaluating the summation (integrations) the equation reduces to.

$$\hbar v_s^* q = [\hbar v_s + \frac{C_1^2 m K_o}{8\pi^2 n v_s M \hbar} + \frac{128\, a^7 [{}^e J(R_h)^2]\, I[(1/2)}{\pi^3 M \hbar v_s^3 (\beta I)^{5/2}}] q \quad \ldots (3.70)$$

v_s^* is the modified velocity of sound. The second term only modifies the velocity of sound. When the third term is calculated, it turns out to be extremely negligible. It is of the order of 10^{-34} ergs. Thus the phonon energy renormalisation due to the phonon-magnon interaction is extremely negligible.

MAGNONS

The magnon Green function $G_{\lambda\lambda}$ is given by

$$\langle\langle a_\lambda, a_\lambda^+ \rangle\rangle = \frac{1}{2\pi} \frac{1}{w - E_\lambda^m - \bar{J}(w,\lambda) - \bar{\phi}(w,\lambda)} \quad (2.58)$$

The pole of this Green function gives the modified energy E_λ^m of the magnons. Hence the distribution function for magnons (neglecting damping) is given by

$$(N_\lambda^m) = \frac{1}{e^{\beta \tilde{E}_\lambda^m} - 1} \quad (3.71)$$

where

$$\tilde{w} = \tilde{E}_\lambda^m = E_\lambda^m + \bar{J}(w,\lambda) + \bar{\phi}(w,\lambda) \quad (3.72)$$

The explicit forms for $\bar{J}(w,\lambda)$ and $\bar{\phi}(w,\lambda)$ are given in equations (2.59) and (2.60). The expression $\bar{J}(w,\lambda)$ is

is obtained by Vonskovaskii, Nagaoka, Potapkov and some other authors[7-11]. The new term $\overline{\phi}(w, \lambda)$ is due to phonon-magnon interaction. It has got pronounced effect on the dispersion relation for magnons. The modification due to electron-magnon interaction i.e. $\overline{J}(w, \lambda)$ is given by

$$\frac{S\,(3\pi^2\,\frac{N_s}{N})^{1/3}\;J_o^2\,a^2\,\lambda^2}{24\pi^2 E_f}$$

N_s is the number of conduction electrons. E_f is the Fermi energy. $\overline{\phi}(w, \lambda)$ is given by (see appendix)

$$\frac{8a^7 q_m^2\,\hbar\,s^2\,[\,{}^{e}J(R_h)]^2}{\pi^2\,I^2\,v_s\,M}\,(\,w - E_\lambda^m\,) \qquad (3.73)$$

Substituting these expressions into the equation (3.72) we get

$$\widetilde{w} = \widetilde{E}_\lambda^m = I^*\lambda^2$$

as follows (and in turn I^* the modified exchange constant after cancelling the common λ^2)

$$I^* = I + \frac{\dfrac{S(\,3\pi^2\,N_s / N\,)^{1/3}\;J_o^2\,a^2}{24\pi^2 E_f}}{[\,1 + \dfrac{8a^7\,\hbar\,s^2\,(\,{}^{e}J(R_h)^2}{\pi^2\,I^2\,v_s\,M}\,]} \qquad (3.74)$$

when we estimate the quantity in the square bracket (by taking standard values mentioned previously) turns out of the order of 1/5. Thus the phonon-magnon interaction modifies the correction due to electron-magnon interaction considerably.

Consequences of the modification of I (Exchange constant)

The modification in I is automatically followed by the modifications in magnetisation and specific heat[12-16].

The change in magnetisation at a given temperature is given by

$$M(t) - M_0 = \Delta M = 2\mu_0 \sum_{\lambda} (N_\lambda^m) \qquad (3.75)$$

Where M_0 is the saturation magnetisation given by $2\mu_0 NS$ and M(t) is the magnetisation at a given temperature equal to $2\mu_0 S_z$. Changing summation to integration we get and taking unit volume

$$\Delta M = \frac{2\mu_0}{(2\pi)^3} \int \frac{4\pi\lambda^2 \, d\lambda}{e^{I^*\lambda^2/k_\beta T} - 1} \qquad (3.78)$$

This reduces to (at low temperatures)

$$\Delta M = 0.117\mu_0 (k_\beta T/I^*)^{3/2} \propto T^{3/2} (I^*)^{-3/2} \qquad (3.79)$$

Since I^* is reduced due to phonon-magnon interaction we will observe increase in $\triangle M$ i.e. change in magnetisation. This result, we have obtained is theoretically correct as the change in magnetisation should increase if we take phonons along with magnons. It will help more in reversing the spins.

Specific heat

The other consequence is the specific heat. We see that the specific heat is enhanced[17]. The internal energy of unit volume of magnon gas is given by

$$E = \sum_{\lambda} E_{\lambda}^{m} (N_{\lambda}^{m}) = \sum_{\lambda} I^* \lambda^2 \frac{1}{e^{I^* \lambda^2 / k_\beta T} - 1} \qquad (3.80)$$

Changing summation to integration we get

$$E \simeq \frac{0.45 (k_\beta T)^{5/2}}{\pi^2 I^{*\,3/2}} \qquad (3.81)$$

Hence the specific heat is proportional to

$$\frac{dE}{dt} = C \propto T^{3/2} (I^*)^{-3/2} \qquad (3.82)$$

As the factor I^* is reduced by a factor of 1/5, the magnon specific heat is enhanced by a factor of 10 % or more[17]

This is to be expected as we consider lattice disorder (phonons) due to lattice vibrations along with the magnons.[18]

Thus the renormalisation of magnon mode due to phonon-magnon interaction is dominant and gives rise to important effects mentioned above.

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C H A P T E R - IV

C H A P T E R - IV

FORMULATION OF OPTICAL PHONON-MAGNON INTERACTION IN FERROMAGNETS

Phenomenological theory of phonon-magnon interaction has been done by some authors[1-4]. Recently it was studied from microscopic point of view[5-7]. The study was confined to (acoustic) phonon-magnon interactions in the case of ferro, anti-ferro and ferri-magnets. It was extended to (optical) phonon-magnon interaction in case of anti-ferro and ferri-magnetic systems[14]. We extend the theory of (optical) phonon magnon interaction in case of ferro-magnetic systems. As we have got ferro-magnetic semiconductors in mind, particularly, optical (phonon-magnon) interaction may become dominant at some stage (range of temperature). The interaction will be formulated by taking into account the mixing of excited and ground state atomic orbitals owing to the crystal field oscillations. These perturbed states when used for formulating the two electron interaction term in the second quantisation representation, furnish the relevant interaction term.

As in equation (2.1), we write the total Hamiltonian of the system as

$$H = H_e + H_m + H_p + H_{em} + H_{ep} + H_{pm} \qquad (4.1)$$

In H_p, H_{ep} and H_{pm} we have to replace acoustic phonons by optical phonons.

$$H_p = \sum_q \hbar\omega_q \left(b_q^+ b_q + 1/2\right) \qquad (4.2)$$

b_q^+ and b_q are the creation and annihilation operators for optical phonons. The form of electron-optical phonon inter action will be that given by Frohlich and others[8-9] in the case of polar semiconductors (continuum model), or one can use the form on the basis of atomistic model.[10-11] We take

$$H_{ep} = \sum_{k,q,\sigma} A_q \left[C^+_{k+q,\sigma} C_{k,\sigma} b_q + C^+_{k-q,\sigma} C_{k,\sigma} b^+_q \right] \quad (4.3)$$

Here A_q is the optical phonon-electron coupling constant given by

$$A_q = 4\pi \left(\frac{e^2 \hbar}{2 \gamma w_q V} \right)^{1/2} \frac{1}{q}$$

where

$$\frac{1}{\gamma} = \frac{w_q^2}{4\pi} \left(\frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_o} \right)$$

ϵ_∞ and ϵ_o are the high frequency and static dielectric constant. w_q is the polar mode frequency.

Interaction Hamiltonian

Following the earlier work[14] we can write the simplified interaction term as below

$$\frac{4}{\sqrt{N}} e^{J(R_h)} g_o \sum_{l,m} \sum_{q} (e^{iq.R_l} + e^{iq.R_m})(b_q^+ + b_{-q}) \underline{S}_{lA} \cdot \underline{S}_{mA} \quad (4.4)$$

where $\underline{S}_l$, $\underline{S}_m$ are the spin operators for the spins at sites l and m respectively. $e^{J(R_h)}$ is the modification of the effective exchange integral owing to the crystal field oscillations (same as defined in Chapter II).

$$g_o = \left(\frac{\hbar}{2M\omega_q} \right)^{1/2}$$

We replace $\underline{S}^+_{lA}$ and $\underline{S}^-_{mA}$ by the usual spin wave representation and use the identity

$$\underline{S}_{lA} \cdot \underline{S}_{mA} = \frac{1}{2} \left(\underline{S}^+_{lA} \underline{S}^-_{mA} + \underline{S}^-_{lA} \underline{S}_{mA} \right) + \underline{S}^z_{lA} \cdot \underline{S}_{mA} \qquad (4.5)$$

$$\left.\begin{aligned} \underline{S}^+_{lA} &= \frac{4S_A}{N} \sum_{\lambda} e^{i\lambda \cdot R_l} a_\lambda \\ \underline{S}^-_{mA} &= \frac{4S_A}{N} \sum_{\lambda} e^{-i\lambda \cdot R_m} a^+_\lambda \end{aligned}\right\} \qquad (4.6)$$

and

$$\underline{S}^z_l = S_A - \frac{2}{N} \sum_{\lambda,\lambda'} e^{i(k-k') \cdot R_l} a^+_\lambda a_{\lambda'}$$

By using equations (4.5) and (4.6) we reduce equation (4.4) to

$$\sum_{\lambda,q} 16 S^2 J(R_h)\, g_o \left(\gamma_{\lambda+q} + \gamma_\lambda - \gamma_q - 1 \right) a^+_\lambda a_{\lambda+q} \left(b^+_q + b_{-q} \right) \qquad \ldots\ldots (4.7)$$

where

$$\gamma_\lambda = \frac{1}{z} \sum_h e^{i\lambda \cdot R_h}$$

and

$$\gamma_q = \frac{1}{z} \sum_h e^{iq \cdot R_h}$$

where

$$R_h = \underline{R}_l - \underline{R}_m$$

The expression ($\gamma_{\lambda+q} + \gamma_{\lambda} - \gamma_{q} - 1$) in equation (4.7) factorises to ($1 + \gamma_{q}$) ($1 - \gamma_{\lambda}$) which reduces to $\lambda^2 a^2$. Here the expansion of γ_{q} and γ_{λ} (exponential function) is used. $R_h \approx a$, in case of a cubic lattice and we have used the approximation

$$q^2 a^2 < 1$$

Using these results the interaction reduces to

$$H_{pm} = \sum_{\lambda, q} \phi_{\lambda q} [a^{+}_{\lambda+q} a_{\lambda} (b_{q} + b^{+}_{-q})]$$

i.e. similar to the equation (2.8) only the factor $\phi_{\lambda q}$ is different and is given by

$$\phi_{\lambda q} = 16 S e_{J(R_h)} (\frac{\hbar}{2Mw_q})^{1/2} a^2 \lambda^2 .$$

This is the modified coupling constant for (optical) phonon magnon interaction. Using this value of coupling constant, we can obtain modified electron-magnon and phonon Green functions and their renormalised energies. The work will be exactly similar to that carried out in Chapters II and III.

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MATHEMATICAL APPENDICES

Appendix I

A) We want to evaluate $\bar{M}_{k\downarrow}$ (Equation 3.31). First we evaluate $\bar{A}(w, k+\lambda\downarrow)$ and $\bar{\phi}(w, k+\lambda)$ involved in the above equation.

$$\bar{A}(w, k+\lambda\downarrow) =$$

$$= \sum_q A_q^2 \left[\frac{[(1+N_q^p) - (n_{k+\lambda-q\downarrow})]}{w-\epsilon_{k+\lambda-q} + E_\lambda^m - E_q^p} - \frac{((N_q^p) - (n_{k+\lambda+q\downarrow})]}{w-\epsilon_{k+\lambda+q}+E_\lambda^m +E_q^p} \right]$$

At low temperatures we take the denominators to be equal and $(N_q^p+1) \simeq 1$, hence

$$\bar{A}(w,k+\lambda\downarrow) = \sum_q \frac{A_q^2}{w-\epsilon_{k+\lambda-q}+E_\lambda^m} \qquad (A.1)$$

Putting $w = \epsilon_k = Ak^2$ and expanding $\epsilon_{k+\lambda-q}$, we get

$$\frac{V\bar{A}_q^2}{4\pi^2} \int \frac{q^3\, dq\, d\mu}{a + b\mu} \qquad (A.2)$$

$\bar{A}_q = A_q q^{1/2}$ and $a = w - \epsilon_{k+\lambda} + E_\lambda^m$, $b = 2Akq$

$\mu = \theta_{kq}$

$$\frac{V\bar{A}_q^2}{8\pi^2 Ak} \int q_m^2 \log\left(\frac{a+b}{a-b}\right) dq \qquad (A.3)$$

If we consider $a < b = 2Akq$ we get(1)

(after putting proper value of $\bar{A}_q$ we finally get)

$$\bar{A}(w,k+\lambda\downarrow) = \frac{C_1^2 mq^2{}_m \left(w-\epsilon_{k+\lambda} + E_\lambda^m \right)}{16\pi^2 nh\epsilon_k \; v_s M} \qquad (A.4)$$

In the same manner $\bar{A}(w,k-\lambda\uparrow)$ can be solved.

B) $\bar{\phi}(w,k+\lambda) =$

$$= \sum_q \phi_{\lambda q}^2 \left[\frac{[(N_{\lambda+q}^m) - (N_q^p)]}{w-\epsilon_{k+\lambda}+E_{\lambda+q}^m - E_q^p} - \frac{((1+N_q^p) + (N_{\lambda+q}^m)]}{w-\epsilon_{k+\lambda} + E_{\lambda+q}^m + E_q^p} \right]$$

..... (B.1)

which reduces to

$$= \sum_q \frac{\bar{\phi}_{\lambda q}^2}{w-\epsilon_{k+\lambda} + E_{\lambda+q}^m} \qquad (B.2)$$

$$= \frac{\bar{\phi}_{\lambda q}^2 V^2}{4\pi^2} \int_0^{q_m} q^3 dq \int \frac{u^2}{a + b\mu} d\mu \qquad (B.3)$$

$$\bar{\phi}_{\lambda q} = \phi_{\lambda q} \lambda^2 q\mu$$

where $a = w - \epsilon_{k+\lambda} + E_\lambda^m$, $b = 2I\lambda q$ and $\mu = \cos\theta_{\lambda q}$

$$\frac{\bar{\phi}_{\lambda q}^2 \, V_\lambda^2}{4\pi^2} \int q^3 \left[\frac{-2a}{4I^2\lambda^2 q^2} + \frac{a^2}{8I^3\lambda^3 q^3} \log \left[\frac{1 + \frac{2I\lambda q}{a}}{1 - \frac{2I\lambda q}{a}} \right] \right] dq \qquad \text{.....(B.4)}$$

expanding the log function we get

$$\frac{-8a^7 q_m^2 \hbar \, (2S) \, e_{J(R_h)}^2}{\pi^2 I^2 v_s M} \left(w - \epsilon_{k+\lambda} + E_\lambda^m \right) \qquad (B.5)$$

In the same manner $\bar{\phi}(w, k-\lambda)$ can be solved.

C) Using the values of $\bar{A}(w, k+\lambda\downarrow)$ and $\bar{\phi}(w, k+\lambda)$ we get

$$\bar{M}_{k\downarrow} = \frac{\sum_\lambda J_m^2(k,\lambda) \left[(n_{k+\lambda\downarrow}) + (N_\lambda^m) \right]}{(w - \epsilon_{k+\lambda} + E_\lambda^m)\left[1 - \frac{c_1^2 m q_m^2}{16n\pi^2 v_s M\hbar} + \frac{8a^7 s^2 \, e_{J(R_h)}^2 q_m^2}{\pi^2 v_s M I^2} \right]} \qquad (C.1)$$

The quantity in the square bracket is independent of λ, hence we first solve

$$\frac{\sum_\lambda J_m^2(k,\lambda) \left[(n_{k+\lambda\downarrow}) + (N_\lambda^m) \right]}{w - \epsilon_{k+\lambda} + E_\lambda^m} \qquad (C.2)$$

Taking $(n_{k+\lambda}) \rightarrow 0$ and $(N_\lambda^m) = e^{-\beta I \lambda^2}$ we get

$$\frac{2SJ_o^2 a^3}{4\pi^2} \int \frac{\lambda^2 e^{-\beta I \lambda^2} \, d\lambda \, d\mu}{-A\lambda^2 - 2Ak\lambda\mu + I\lambda^2} \qquad (C.3)$$

$$\mu = \cos \theta_{\lambda q}$$

$$\frac{-2SJ_o^2 a^3}{8\pi^2 Ak} \int e^{-\beta I \lambda^2} \log \frac{\lambda + 2k}{\lambda - 2k} \, d\lambda \qquad (C.4)$$

Using $\lambda < k$ assumption we get.

$$= -\frac{2SJ_o^2 a^2}{8\pi^2 Ak^2} \int \lambda^2 e^{-\beta I \lambda^2} \, d\lambda \qquad (C.5)$$

Putting $\sqrt{\beta I}\,\lambda = x$, we get finally

$$= \frac{2SJ_o^2 a^2 \sqrt{\pi}}{16\pi^2 \epsilon_k (\beta I)^{3/2}} \qquad (C.6)$$

Using the results (A.6), (B.5) and (C.6) we get

$$\frac{\dfrac{-SJ_o^2 a^3 \sqrt{\pi}}{16\pi^2 \epsilon_k (\beta I)^{3/2}}}{\left[1 - \dfrac{C_1^2 m q_m^2}{16n\pi^2 v_s M\hbar} + \dfrac{8a^7 \hbar s^2 \, e_{J(R_h)}{}^2}{\pi^2 v_s M I^2}\right]} \qquad (C.7)$$

In the same manner we can get

$$\bar{M}_{k\uparrow} = \frac{-\frac{8J_o^2 a^3 \sqrt{\pi}}{16\pi^2 \epsilon_k (\beta I)^{3/2}} - \frac{8J_o^2 a^3 \lambda_m^3}{12\pi^2 \epsilon_k}}{\left[1 - \frac{C_1^2 m q_m^2}{16n\pi^2 v_s M\hbar} + \frac{8a^7 \hbar S^2 e_{J(R_h)}^2}{\pi^2 v_s M I^2}\right]} \qquad (C.8)$$

D) In order to evaluate $\bar{P}_{k\uparrow}$ and $\bar{P}_{k\downarrow}$, we have to evaluate $\bar{J}(w,k+q\downarrow\downarrow)$, $\bar{J}(w,k-q\downarrow\downarrow)$ and $\bar{\phi}(w,k\pm q)$.

$$\bar{J}(w,k+q\downarrow) = \frac{\sum_\lambda J_m^2(k,\ )\left[(N_\lambda^m) + (n_{k+q+\lambda\downarrow})\right]}{w - \epsilon_{k+q+\lambda} + E_\lambda^m + E_q^p} \qquad (D.1)$$

Proceeding in the way followed in appendix C, we get

$$= \frac{-J_o^2 8a^3 m\ (\ w-\epsilon_{k+q} + E_q^p\ )}{4\pi^{3/2} \epsilon_k \hbar \sqrt{\beta I}} \qquad (D.2)$$

Similarly $\bar{J}(w,k-q\uparrow)$ is given by

$$\left[\frac{-J_o^2 8a^3 m}{4\pi^{3/2} \hbar^2 \epsilon_k \sqrt{\beta I}} - \frac{-J_o^2 8a^3 m \lambda_m}{4\pi^2 h^2 \epsilon_k}\right](w-\epsilon_{k+q} + E_q^p\ ) \qquad (D.3)$$

E) Now we calculate $\bar{\phi}(w,k \pm q)$

$$\bar{\phi}(w,k \pm q) = \frac{\sum_\lambda \phi_{\lambda q}^2 \left[(N_{\lambda+q}^m\ ) - (N_\lambda^m\ )\right]}{w - \epsilon_{k+q} + E_{\lambda+q}^m - E_\lambda^m} \qquad (E.1)$$

It can be written as

$$\sum_{\lambda} \phi^2_{\lambda q} \left[\frac{(N^m_\lambda)}{w-\epsilon_{k+q}+E^m_\lambda - E^m_{\lambda-q}} - \frac{(N^m_\lambda)}{w-\epsilon_{k+q}+ E^m_{\lambda+q}-E^m_\lambda} \right] \qquad (E.2)$$

$$\frac{\overline{\phi}^2_{\lambda q}\ q\ V}{4\pi^2} \int \lambda^4 d\lambda \mu^2\ d\mu \quad [\text{ Sq. bracket given above}) \qquad (E.3)$$

Considering the first term only of (E.3)

$$\frac{\overline{\phi}^2_{\lambda q}\ q\ V}{4\pi^2} \int \lambda^4\ e^{-\beta I \lambda^2} d\lambda \int_{-1}^{1} \frac{\mu^2}{1 + b\mu}\ d\mu$$

where $a = w - \epsilon_{k+q} - Iq^2$, $b = 2 I\lambda q$ and $\mu = \cos \theta_{\lambda q}$

We have[1]

$$\int_{-1}^{1} \frac{\mu^2}{a + b\mu}\ d\mu = \frac{-2a}{b^2} + \frac{a^2}{b^3} \log \frac{a + b}{a - b}$$

We take $b < a$ and use the expansion

$$\log \frac{(1 + x)}{(1 - x)} = 2\left(x + \frac{x^3}{3} + \ldots\ldots \right)$$

where $x = a/b$. We finally get

$$= \frac{\overline{\phi}^2_{\lambda q}\ V\ q}{12\pi^2 a} \int 2\lambda^4\ e^{-\beta I \lambda^2} d\lambda \qquad (E.4)$$

$$\frac{\bar{\phi}^2_{\lambda q}\ V\ q\ \Gamma(5/2)}{12\pi^2\ (\beta I)^{5/2}} \qquad \frac{1}{a} \qquad \text{(E.5)}$$

After solving for the second term we finally get,

$$\bar{\phi}(w,k+q) = \frac{\bar{\phi}^2_{\lambda q}\ V\ q\ \Gamma(5/2)}{12\pi^2\ (\beta I)^{5/2}} \left(\frac{1}{a} - \frac{1}{a'} \right) \qquad \text{(E.6)}$$

where $a' = w - \epsilon_{k+q} + I_q^2$

Substituting for a and a'

$$\frac{1}{a} - \frac{1}{a'} = \frac{3}{2} \frac{I}{A\epsilon_k}$$

Hence

$$\bar{\phi}(w,k+q) = \left[\frac{32a^7\ e_J(R_h)^2\ s^2\ \mathrm{Im}\ \Gamma(5/2)}{\pi^2\ v_s\ N\hbar\epsilon_k\ (\beta I)^{5/2}} \right] q \qquad \text{(E.7)}$$

When we estimate this quantity, it is extremely negligible (which can be verified by substitting relevant values). Knowing all these quantities, we can evaluate $\bar{P}_{k\uparrow}$ and $\bar{P}_{k\downarrow}$. Also the factors $\bar{J}(w,\ k \pm q,\ \sigma)$ modify the denominator by a constant factor, we have to evaluate

$$\sum_q A_q^2 \left[\frac{[(N_q^p) + (n_{k+q,\sigma})]}{w - \epsilon_{k+q} + E_q^p} + \frac{[(1+N_q^p) - (n_{k-q,\sigma})]}{w - \epsilon_{k-q} - E_q^p} \right]$$

..... (F.1)

This can be evaluated on the same lines as we have evaluated $\bar{A}(w, k+\lambda)$ etc. Thus the above factor reduces to

$$= \frac{- C_1^2 \hbar q_m^4}{64\pi^2 n M v_s \epsilon_k} \qquad (F.2)$$

Substituting the values of $\bar{J}(w, k \pm q\downarrow\uparrow)$, we get [$\bar{\phi}(w, k+q)$ is negligible]

$$\bar{P}_{k\uparrow} = \frac{\dfrac{- C_1^2 \hbar q_m^4}{64\pi^2 nM v_s \epsilon_k}}{\left[1 - \dfrac{J_o^2 S a^3 m}{4\pi^{3/2} \hbar^2 \epsilon_k \sqrt{\beta I}}\right]} \qquad (F.3)$$

and

$$\bar{P}_{k\downarrow} = \frac{- C_1^2 \hbar q_m^4}{64\pi^2 n \epsilon_k v_s M \left[1 - \dfrac{J_o^2 S a^3 m}{4\pi^{3/2} \hbar^2 \epsilon_k \sqrt{\beta I}} - \dfrac{J_o^2 S a^2 m_{\lambda_m}}{4\pi^2 \hbar^2 \epsilon_k}\right]} \qquad (F.4)$$

Appendix - II

The renormalised phonon-energy is given by

$$\tilde{E}^{p}_{q} = E^{p}_{q} + \bar{A}(w, q) + \bar{\phi}(w, q)$$

The second term is already obtained. We will try to evaluate $\bar{\phi}(w, q)$ (Equation 2.44)

$$\bar{\phi}(w,q) = \frac{\sum_{\lambda} \phi^{2}_{\lambda q} \left[(N^{m}_{\lambda - q}) - (N^{m}_{\lambda}) \right]}{w + E^{m}_{\lambda - q} - E^{m}_{\lambda}}$$

The integration is performed in the same manner as we have done for $\bar{\phi}(w, k \pm q)$ (appendix D). Only we have to replace w by $\hbar v_s q$ in this case. Hence we get

$$\bar{\phi}(w,q) = \frac{128\, a^{7}\, e_J (R_h)^{2}\, S^{2}\, I\, \Gamma(5/2)}{\pi^{2}\, M\, \hbar\, v_s^{3}\, (\beta I)^{5/2}}$$

$$\simeq \quad bT^{5/2}$$

when we estimate 'b', it turns out to be extremely negligible.

$\bar{\phi}(w,q)$ in case of (optical) phonon-magnon interaction is also extremely negligible.

Appendix - III

The renormalised magnon energy is given by

$$\tilde{E}^{m}_{\lambda} = E^{m}_{\lambda} + \bar{J}(w,\lambda) + \bar{\phi}(w,\lambda)$$

The second term has been already obtained (Vonskovaski et al). We will try to evaluate $\bar{\phi}(w,\lambda)$ (Equation 2.60) This can be solved in exactly similar manner as we have solved $\bar{\phi}(w,k \pm \lambda)$ (Appendix 1B). Hence finally, we get

$$\bar{\phi}(w,\lambda) = \frac{8a^{7} q_{m}^{2} \hbar (2S)^{2} e_{J(R_h)}^{2}}{\pi^{2} I^{2} v_{s} M} (w - E^{m}_{\lambda})$$

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S Y N O P S I S

Interactions Involving Elementary Excitations in Ferromagnetic Semiconductors

In some magnetic semiconductors one can visualise the presence of all three types of quasi particles i.e. electrons, phonons and magnons. While studies have been made on electron-phonon and electron-magnon interactions separately in different kinds of solids, there does not seem to be any report of detailed investigations where the simultaneous interactions of electron-phonon, electron-magnon and magnon-phonon are taken into account. Such studies seem to be worthwhile in view of recent discoveries of some ferromagnetic semiconductors (e.g. doped $CdCr_2Se_4$ etc.).

Chapter I

In this chapter we have reviewed important concepts leading to various mechanisms of magnetic coupling in solids. The properties of the two principle elementary excitations i.e. phonons and magnons are given in detail. After elucidating the three interactions i.e. electron-phonon, electron-magnon and the phonon-magnon we have discussed the necessity of including the third interaction.

In the appendix, we have discussed the Green functions technique which is very effective in many body problems. Once these Green functions are known various physical quantities of

the system such as energy, relaxation time, etc. can be readily calculated.

Chapter II

In the second chapter we have formulated the total Hamiltonian of the system, by taking into account the simultaneous presence of phonons and magnons in a magnetic crystal. We have defined the electron, phonon and magnon Green functions and after solving the equations of motion, we have obtained explicit expressions for electron, phonon and magnon Green functions respectively. These are completely new expressions which incorporate the effect of phonon-magnon interaction for the first time.

Chapter III

Section I

We have discussed here the properties of the dressed electrons. The power and simplicity of Green function technique becomes clear here. The poles of the Green functions give the renormalised energies of particle or excitation and imaginary part gives the magnitude of damping. Contrary to electron phonon and electron-magnon interactions the phonon-magnon interaction has a tendency to increase the energy of the electron and hence reduce the effective mass of the electron.

Section II

The Green function method for obtaining the relaxation time is given. By this method (without considering phonon-magnon

interaction) we have obtained the expression for mobility i.e. $T^{-3/2}$ dependence in case of semiconductors. Also the expression for mobility at low temperature is discussed.

Section III

In this section the renormalised energies of phonons and magnons are obtained. The modification of phonon energy is extremely negligible. But the modification in the case of magnon is very much appreciable. By taking into account the renormalised energy, the modified exchange constant is calculated. It is shown that the exchange constant is reduced, which gives rise to enhancement in specific heat and magnetisation. The enhancement of specific heat is predicted for the first time.

Chapter IV

In this chapter we have formulated the theory of optical (phonon-magnon) interaction. An expression for coupling constant is obtained. With this value of coupling constant we carry out other calculations similar to the (acoustic) phonon-magnon interaction case.

Chapter V

This consists of mathematical appendices. The study of these appendices show that we have to limit ourselves to temperature and wave-length range in order to simplify the calculations.